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SUMMARY

The objective of this study is to get for several food products the kind of demand information that is relevant when a price support administrator selects a support price. The specific problems dealt with are those of estimating the retail demand conditions for (1) pork, beef and poultry products and (2) eggs.

In connection with each problem, demand equations have been set up. In each case demand relations have been estimated by the least squares method, which treats demand equations in isolation from supply equations. Demand equations have also been estimated by simultaneous equations methods, which deal with a demand equation as part of a system of equations including a supply equation and perhaps others.

In the pork, beef and poultry products case the most reasonable results appear to be those of a simultaneous equations system. The estimates include the following:

A 1 percent rise in the price of pork can be expected to bring a 0.91 percent fall in the quantity of pork sold at retail, and a 1 percent rise in disposable personal income can be expected to bring a 0.76 percent increase in the quantity of pork sold at retail.

A 1 percent rise in the price of beef can be expected to bring a 0.77 percent fall in the quantity of beef sold at retail, and a 1 percent rise in disposable personal income can be expected to bring a 0.65 percent rise in the quantity of beef sold at retail.

A 1 percent rise in the price of poultry products can be expected to bring a 0.68 percent fall in the quantity of poultry products purchased at retail, and a 1 percent rise in disposable personal income can be expected to bring a 0.53 percent rise in the quantity of poultry products purchased at retail. There is also an estimate of the influence of the price of each meat on the retail sales of the other meats.

The least squares method appears to give the most reasonable results for eggs: A 1 percent rise in the price of eggs can be expected to bring a 0.55 percent fall in the quantity of eggs purchased at retail, and a 1 percent rise in disposable personal income can be expected to bring a 0.41 percent rise in the quantity purchased at retail. The influences of

other factors affecting the retail sales of eggs are also examined.

If the support price of any meat is effective, raising it will raise the producer's total revenue. This is true because the elasticity of the quantity of each meat with respect to its own price is less than 1.

When there are price support operations the relative prices of the meats will probably change. On the basis of the effect of the price of each meat on the quantities of the others, it appears that relative price changes will probably leave poultry consumption more nearly constant than the consumption of either pork or beef.

In the case of each meat, quantity is inelastic with respect to current income. The influence of income on poultry consumption appears to be less than on either pork or beef.

Application of Econometric Procedures to the Demands for Agricultural Products[1]

By J. A. Nordin, George G. Judge and Omar Wahby

A legislator or administrator concerned with the area of price supports is concerned with a problematic situation—a situation in which there is some question about the course of action to be taken on the basis of the objectives thought to be held by the voters. In connection with this problematic situation a number of individual problems arise. Each problem may deal with the reactions to a given course of action in one of the sectors of the economy. Thus predicting consumers' responses to selected economic changes is one of the problems that arise in the exploration of the problematic situation associated with price supports.

The present bulletin deals with the following problems: (1) What variables determine the quantities of retail sales of pork, beef and poultry products, and how are the determining variables related to these quantities? (2) What variables determine the quantity of retail sales of eggs, and how are the determining variables related to this quantity?

The Agricultural Adjustment Administration bought cattle and hogs from May 1933 to Dec. 31, 1934. Pork and eggs were included in the list of commodities subject to the first food stamp plan in 1939. The Steagall amendment of 1941 set up mandatory price support provisions for hogs, chickens, eggs and turkeys. During and immediately after World War II support action was not needed for most goods. The price of hogs was supported in 1943-44, and the price of eggs in 1944. As of June 1954, the government was not supporting the price of any of the goods whose demand conditions are examined in the present study. But the Agricultural Act of 1949 provides that any of these prices may be supported at from 0 to 90 percent of parity at the discretion of the Secretary of Agriculture.

GENERAL PROCEDURE IN DEALING WITH SELECTED PROBLEMS

Whoever helps a price support administrator deal with his problematic situation is helping him choose an action appropriate to the voters' objectives in the price support area.

[1] Project 1091 of the Iowa Agricultural Experiment Station. For suggestions and criticism the authors are indebted to Martin Beckman, John Gurland, Earl O. Heady, O. Kempthorne, Geoffrey Shepherd and Gerhard Tintner.

The choice of action depends partly on the answers to problems like the ones with which we are concerned. In attacking a given problem, the investigator sets up a tentative answer that appears reasonable to him. Such a tentative answer we shall call a hypothesis. The investigator then makes tests to determine whether it would be prudent to act as though the tentative answer were the correct one. That is, the investigator devises a test for the hypothesis.[2]

In the physical sciences it is usual to set up a hypothesis and then design a crucial experiment to help the investigator decide whether it is prudent to act as though the hypothesis were correct. In the social sciences deliberate experiment is usually impossible. We must proceed as though we had to analyze the results of poorly designed experiments.

The hypothesis in demand analysis is the contingent predictions of the values of various economic variables. We shall begin with a *model*—a list of variables relevant to the problem, together with the forms of the equations in which the variables are said to be related. Then on the basis of the variables in a base period we shall estimate the coefficients which specify the way in which the variables are related to each other. After the coefficients have been incorporated into the model, it serves as a mechanism for forecasting the values of certain variables in a prediction period after the base period, given the values of other variables in the base period and/or the prediction period. We shall deal only with linear models since they offer considerable advantage in terms of simplicity of handling. Of course the variables may be in logarithmic form.

In the construction of a hypothesis we may impose certain restrictions on our model. For instance, we may be reasonably sure that a certain coefficient must lie between 0 and 1. When we use the data from our base period, this restriction will limit the way in which the data will condition our predictions. Any proposition (such as the proposition that the value of a coefficient must lie between 0 and 1) used in an investigation, but not tested by the investigation, will be called an *assumption*. Thus the word assumption may include propositions that are sometimes called "axioms" or "postulates."

The hypothesis is tested when we compare its predictions with the observed values of the predicted variables. If the predictions are sufficiently accurate, we shall consider it pru-

[2] The concept of a statistical hypothesis is sometimes used in a more restricted sense.

dent to act as though the hypothesis were correct. Deciding whether the predictions are sufficiently accurate may involve some perplexing questions; this issue will be dealt with later.

SPECIFIC PROCEDURE IN DEALING WITH SELECTED PROBLEMS

We shall try to predict retail demand conditions on the basis of time series. The quantity that an individual consumer will buy in the market for a given good in a given time period will depend on the price of the good. His income in the current period and in past periods will also influence the result.[3]

The prices of other goods will also be relevant. We may take account of the individual prices of goods closely related to the good in question, either as substitutes or as complements. If computational troubles were of no consequence, it would undoubtedly be worthwhile to treat a great many other commodities in this way. However, we must consider expense, so at some point we stop treating other goods individually and lump them together by using an index of prices.

The effects of changing tastes are taken into account by making "time" (the lapse of time during the period to which the study applies) a variable in one form or another.

Throughout the procedure using time series the central idea is that of prediction. We are interested in historical associations only insofar as they assist us in making predictions about the future level of the variable we are interested in.

LEAST SQUARES

Most empirical time series work has been done by means of the least squares single equation method.[4] In this method the variable to be predicted is considered "dependent" and

[3] See James Duesenberry, Income, saving, and the theory of consumer behavior. Harvard University Press, Cambridge, 1949, pp. 76, ff. Duesenberry concentrates on the individual's highest past income. However, the influence of this highest past income depends partly on the time distance by which it is removed from the present period, and also on the degree to which the individual is convinced that the high income is to be unique in his experience.

[4] See, for instance, A. R. Prest, Some experiments in demand analysis, 31 Rev. of Econ. and Statistics 33, February 1949; Geoffrey Shepherd, Changes in the demand for meat and dairy products in the United States since 1910, Iowa Agr. Exp. Sta. Res. Bul. 368. 1949; Richard Stone, The role of measurement in economics, Cambridge University Press. 1951.

the variables contributing to the prediction process are considered "independent." The worker uses all the relevant information that his resources will permit him to use. The equations used are of the type

$$(1) \quad x_1 = a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n$$

where x_1 is the dependent variable, x_2 through x_n the independent variables and the a's constants to be determined by the least squares method. The a's are determined so that the sum of squares of differences between calculated and observed values of x_1 is a minimum. Equation (1) is a regression equation, and the a's are regression coefficients.

A number of objections have been raised against this procedure in connection with analysis of economic time series. In the first place, it has been objected that, in a special sense, the regression equation is arbitrary. Suppose we wish to predict a series x_1 on the basis of a series x_2, where each series has a mean of zero. We must decide whether to minimize the sum of squares of deviations of observations from our prediction line in the x_1 direction or in the x_2 direction. If we minimize in the x_1 direction, a one-unit increase in x_2 will be expected to cause a $\frac{\Sigma x_1 x_2}{\Sigma x_2^2}$-unit increase in x_1. But if we minimize in the x_2 direction, a one-unit increase in x_2 will be expected to cause a $\frac{\Sigma x_1^2}{\Sigma x_1 x_2}$-unit increase in x_1. If *n* variables are involved, there are *n* possible directions in which the minimization could be made. Thus there are *n* different regression functions. If each of them is solved for x_1, there will be *n* different coefficients showing the relation between x_1 and x_2.

This objection does not appear to be serious. We have undertaken the inquiry in order to improve our ability to predict the future values of one of the variables involved. Then it is reasonable to assume that we wish to minimize the seriousness of errors in predicting *this* variable, rather than any other. Thus we choose the direction of minimization when we choose the variable to be predicted.

A second objection, raised by Ragnar Frisch, is related to the possibility of getting meaningless values for the a's.[5] Suppose that each of two independent variables consists of two parts—a "systematic" part and an "error" part. If the

[5] Ragnar Frisch, Statistical confluence analysis by means of complete regression systems, Universitetets Oekonomiske Institutt, Oslo, 1934.

systematic parts are perfectly correlated and the errors are zero, it will be impossible to get unique values for the a's—although this difficulty does not reduce our ability to predict the value of the independent variable. If the systematic parts are perfectly correlated and the errors are not zero, then the a's can be calculated and their values will depend only on the errors. This general situation is given the name *multicollinearity,* since we assume one linear regression (the equation used to predict x_1) while there is at least one other linear relation in the problem (the linear relation between the systematic parts of two or more independent variables).

If the systematic parts are not perfectly correlated, then the a's will not depend exclusively on the errors. But the errors may be large enough to make the variance of the predicted values dangerously large.

A third objection relates to the possibility of getting "nonsense correlations." Suppose that every economic series trends strongly upward over time. If we run the usual multiple regression we find that we can apparently explain any of the series fairly well on the basis of the variations in any combination of the other series. However, a simple regression with time as the independent variable might give us nearly as complete an explanation as we get by using the independent variables we have selected.

One method of dealing with this trouble is to introduce time as one of the variables. Then the regression coefficient associated with any of the other independent variables shows the effect of that variable when time is held constant. We seem to be close to the multicollinearity issue again—but it is very unlikely that any of the independent variables other than time will be perfectly correlated with time.

Another way of dealing with this difficulty involves the use of difference equations. We replace x_i (j) (the value of x_i during the j^{th} interval) with x_i (j) — αx_i (j — 1), where α is a parameter to be estimated. If the effect of whatever variables we group under the name "time" is linear, then taking first differences as indicated above will remove the influence of time.[6] If the influence of time is more complicated, then part of it will remain after we have taken first differences for all the variables. In some cases it may be necessary to introduce variables with lags of more than one time unit.

[6] That is, the influence of time will appear in the constant term.

However, using first differences involves the assumption that the lagged values of the variables are not interesting on their own account. For instance, if x is income, we may not be willing to say that the significance of this period's income depends only on its excess or deficiency in relation to the income of the last period. If both the incomes are high relative to the incomes of most periods, we may cover up the most essential income effects if we deal only with the first differences of incomes.

The fourth objection to the single equation least squares procedure is the most recent and the most serious. This objection comes from the staff of the Cowles Commission, and the most complete analysis based on the objection is included in Cowles Commission Monograph No. 10, Statistical Inference in Dynamic Economic Models (John Wiley and Sons, New York, 1950).[7] Even though we may be interested in predicting the future values of just one variable, it is said that this variable is determined jointly with a large number of other variables. It is said that in general we must not deal with just one equation; we must set up a complete model, in which the number of equations is the same as the number of variables regarded as being jointly determined by the working of the model. The basis for this objection will be considered below in connection with the simultaneous equations method.

SIMULTANEOUS EQUATIONS

In this section we shall present a simplified exposition of the simultaneous equations procedure in estimating economic relations.[8] To avoid unnecessarily complicating the presentation, we shall deal with a two-equation system. The extension to three or more equations is relatively easy; the rules will be stated in terms that are applicable to the more general case.

Let us assume that we wish to investigate the determination of price and quantity in a market. We have one demand equation and one supply equation. We assume that these two equations show the only relations that are relevant in

[7] See also Cowles Commission Monograph No. 14, Studies in econometric method. John Wiley and Sons, New York. 1953.

[8] The exposition is based on T. W. Anderson and Herman Rubin, Estimation of the parameters of a single equation in a complete system of stochastic equations, 20 Annals of Mathematical Statistics 46. March 1949.

the determination of the values of the economic variables appearing in the model.

We think of the market price and quantity as being determined by the requirement that they must satisfy both the demand and the supply equation. These equations are called structural equations. We shall also refer to them as *basic equations*—they specify the basic economic relations that underlie the operation of a segment of the economy. The coefficients in a basic equation are called *basic coefficients.*

Some variables in the model are considered to be determined outside the operation of the model: They are called *exogenous* variables. For instance, a supply equation may include a variable related to the state of the weather; this variable is not thought of as being determined by the interactions within the model. The variables that are considered determined by the interactions within the model are called *endogenous.*

Sometimes the lagged values of an endogenous variable appear in a model. Thus we may have a demand equation relating today's price and quantity to income and yesterday's price. Yesterday's price is a lagged endogenous variable. Of course, it is not considered as being determined within the model used to explain the determination of today's price and quantity. It is convenient to group lagged endogenous variables with exogenous variables under the heading *predetermined* variables. Current endogenous variables are given the name *jointly determined*; they are the variables whose values are considered jointly determined by the interaction among the variables in the equations of the model.

It will be assumed that each jointly determined variable and each predetermined variable is observed without error.[9] However, it will also be assumed that there are some *disturbances*—some random variables that influence the interaction process but are not directly observed. There will be one disturbance in each basic equation. Each disturbance has an expected value of zero. Within one equation the values of the disturbance term at various time intervals are assumed to be independent of each other. Within each basic equation the disturbance term is assumed to be independent of every predetermined variable in the basic equation.

[9] Models having disturbances but no errors of observation are called shock models; models having errors of observation but no disturbances are called error models. The models appearing in the present discussion are shock models.

In order to see the relation between least squares procedure and simultaneous equations procedure, let us consider the following two-equation model:

$$(2)\quad p + \alpha q = \zeta_1$$

$$(3)\quad q + \delta I = \zeta_2,$$

where p is price, q is quantity, I is income, ζ_1 and ζ_2 are disturbances and α and δ are basic coefficients. The variables p and q are considered to be determined by the joint actions described in equations (2) and (3). I is considered as being determined outside this two-equation system.

By selecting the form of each equation we have made some assumptions about the way in which the variables interact. (2) is a demand equation, showing the interrelation between price and quantity in the minds of the buyers. (3) is a supply equation, showing the quantity supplied depending on just income and disturbances. When we have estimated α and δ we have decided how we shall predict future values of price and quantity.

Let us concentrate on the estimation of α. The objective of this section is to show that using least squares procedure on equation (2) alone may give us an estimate that lacks consistency—that fails to converge in probability to the true value of α as the sample size approaches infinity.

Let us use the least squares procedure to estimate α just as though we had only equation (2) to work with. First (2) is solved for p, so that

$$(4)\quad p = -\alpha q + \zeta_1.$$

We consider p the dependent variable, in the least squares sense, and q the independent variable. We get the estimate

$$(5)\quad \alpha^* = -\frac{\Sigma pq}{\Sigma q^2}.$$

The possibility of lack of consistency in the least squares result shows up when we compare the least squares estimate with the estimate we get by dealing with equations (2) and (3) together. Combining the two equations we have

$$(6)\quad p = \alpha\delta I + \zeta_1 - \alpha\zeta_2.$$

This equation involves only one jointly determined variable, p. I is independent of $(\zeta_1 - \alpha\zeta_2)$. Therefore it is legitimate to use least squares procedure to estimate $\alpha\delta$:

$$(7) \quad (\widehat{\alpha\delta}) = \frac{\Sigma pI}{\Sigma I^2}.$$

Similarly it is legitimate to use least squares estimation in equation (3). We have

$$(8) \quad \hat{\delta} = -\frac{\Sigma qI}{\Sigma I^2}.$$

From (7) and (8) we have a consistent estimate of α:

$$(9) \quad \hat{\alpha} = -\frac{\Sigma pI}{\Sigma qI}.$$

It is apparent that (5) and (9) are not equivalent expressions; I enters into (9) directly, but does not enter into (5) directly. Thus it is not to be expected that using least squares procedure on (2) will give us a consistent estimate of α.

UNIQUENESS (IDENTIFICATION)

Still confining our attention to two-equation models, let us next consider certain complications that may arise in using simultaneous equations to estimate the basic coefficients. We wish to know whether we can find a unique value for a given basic coefficient. In an unfavorable case many different values of a given basic coefficient may be compatible with the observed values of the jointly determined variables and the predetermined variables. If this is the case, we say that the basic coefficient is not identified or is not uniquely determined. We shall see the conditions under which such a basic coefficient is uniquely determined.

THE UNIQUELY DETERMINED (JUST IDENTIFIED) CASE

Consider the following model:

$$(10) \quad \text{(Demand equation)} \quad p + \alpha q = \zeta_1$$

$$(11) \quad \text{(Supply equation)} \quad \eta p + q + \delta I = \zeta_2.$$

Assume that (10) is the *chosen equation*—the basic equation whose coefficients we wish to estimate. (In the present case, of course, there is only α to estimate.)

For the reason just suggested we cannot use the least squares procedure in estimating α. In this case we cannot even use it in estimating η, since both p and q appear in (11). But (10) and (11) are two linear equations in p and q. So we can solve them for p and q in terms of the other variables:

$$(12) \quad p = \frac{\alpha\delta}{1 - \alpha\eta} I + \frac{\zeta_1 - \alpha\zeta_2}{1 - \alpha\eta} \quad \text{and}$$

$$(13) \quad q = -\frac{\delta}{1 - \alpha\eta} I + \frac{\zeta_2 - \eta\zeta_1}{1 - \alpha\eta}.$$

Equations (12) and (13) are called reduced form equations. We shall also refer to them as *isolated form equations,* since one jointly determined variable is isolated in each equation—p is isolated in (12) and q in (13).

The isolated form coefficients can be estimated by least squares since there is only one jointly determined variable in each isolated form equation. In each case we wish to estimate the coefficient of the predetermined variable I. The estimates are:

$$(14) \quad \left[-\frac{\hat{\delta}}{1 - \alpha\eta}\right] = \frac{\Sigma qI}{\Sigma I^2}, \quad \text{and}$$

$$(15) \quad \left[\frac{\hat{\alpha\delta}}{1 - \alpha\eta}\right] = \frac{\Sigma pI}{\Sigma I^2}.$$

By dividing (15) by (14) we get

$$(16) \quad \hat{\alpha} = -\frac{\Sigma pI}{\Sigma qI}.$$

Here $\hat{\alpha}$ is said to be uniquely determined. It is subject to sampling variations, of course. But there is just one indication of its value on the basis of a given sample. Equation (10) is also said to be uniquely determined, since each of its basic coefficients is uniquely determined. For the time being we shall not set up a rule for recognizing equations that are uniquely determined. But notice that there are two jointly determined variables in the chosen equation (equation (10)), and that there is just one predetermined variable (I) that appears in the model but does not appear in the chosen equation. The number of jointly determined variables appearing in the chosen equation is one greater than the number of predetermined variables appearing in the model, but not appearing in the chosen equation.

THE INDETERMINATE (UNIDENTIFIED) CASE

A basic equation is indeterminate if any of its coefficients is indeterminate. Equation (11) is such an equation. Again we get equations (14) and (15) as the legitimate least squares estimates of the coefficients of I in the two isolated form equations, (12) and (13). But it is not possible to solve for either $\hat{\eta}$ or $\hat{\delta}$. All we can get is the equation

$$(17) \quad -\hat{\delta}\frac{\Sigma I^2}{\Sigma qI} = 1 + \hat{\eta}\frac{\Sigma pI}{\Sigma qI}.$$

An infinite number of combinations of $\hat{\eta}$ and $\hat{\delta}$ will satisfy (17). Therefore we may say that both $\hat{\eta}$ and $\hat{\delta}$ are indeterminate, and that equation (11) is indeterminate (unidentified).

In the uniquely determined case the number of jointly determined variables appearing in the chosen equation was one greater than the number of predetermined variables appearing in the model but not appearing in the chosen equation. In the present indeterminate case the former number exceeds the latter by 2.

THE OVERDETERMINED (OVERIDENTIFIED) CASE

In the overdetermined case we have two or more estimates of a basic behavior coefficient, but the number of estimates is finite. We are not free to choose just any value for the coefficient as we were in the indeterminate case. But we do not have a unique estimate as in the uniquely determined case.

Consider the model

$$(18) \quad \text{(Demand equation)}\ p + \alpha q = \zeta_1$$

$$(19) \quad \text{(Supply equation)}\ \eta p + q + \delta I + \epsilon C = \zeta_2,$$

where C is an index of production cost. We wish to estimate α.

The isolated form equations are

$$(20) \quad q = -\frac{\delta}{1-\alpha\eta} I - \frac{\epsilon}{1-\alpha\eta} C + \frac{\zeta_2 - \eta\zeta_1}{1-\alpha\eta} \qquad \text{and}$$

$$(21) \quad p = \frac{\alpha\delta}{1-\alpha\eta} I + \frac{\alpha\epsilon}{1-\alpha\eta} C + \frac{\zeta_1 - \alpha\zeta_2}{1-\alpha\eta}.$$

The least squares estimates of the coefficients of I and C in (20) are

$$(22)\ \left[-\frac{\delta}{1-\alpha\eta}\right]' = \frac{\Sigma qI\ \Sigma C^2 - \Sigma qC\ \Sigma IC}{\Sigma I^2\ \Sigma C^2 - \Sigma IC\ \Sigma IC} \qquad \text{and}$$

$$(23)\ \left[-\frac{\epsilon}{1-\alpha\eta}\right]' = \frac{\Sigma I^2\ \Sigma qC - \Sigma qI\ \Sigma IC}{\Sigma I^2\ \Sigma C^2 - \Sigma IC\ \Sigma IC}.$$

Next let us estimate the coefficients of I and C in (21) in the same way:

$$(24)\ \left[\frac{\alpha\delta}{1-\alpha\eta}\right]' = \frac{\Sigma pI\ \Sigma C^2 - \Sigma pC\ \Sigma IC}{\Sigma I^2\ \Sigma C^2 - \Sigma IC\ \Sigma IC} \qquad \text{and}$$

$$(25)\ \left[\frac{\alpha\epsilon}{1-\alpha\eta}\right]' = \frac{\Sigma I^2\ \Sigma pC - \Sigma IC\ \Sigma pI}{\Sigma I^2\ \Sigma C^2 - \Sigma IC\ \Sigma IC}.$$

When we divide (24) by (22) we get

$$(26) \qquad \alpha' = -\frac{\Sigma pI\ \Sigma C^2 - \Sigma pC\ \Sigma IC}{\Sigma qI\ \Sigma C^2 - \Sigma qC\ \Sigma IC}.$$

But when we divide (25) by (23) we get

$$(27) \qquad \alpha^* = -\frac{\Sigma I^2\ \Sigma pC - \Sigma IC\ \Sigma pI}{\Sigma I^2\ \Sigma qC - \Sigma qI\ \Sigma IC}.$$

Since (26) and (27) are not equivalent estimates of α, α is overdetermined. Two operations based on the isolated form equations lead to contradictory requirements for α.

Let H denote the number of jointly determined variables appearing in the chosen equation, and D denote the number of predetermined variables appearing in the model but not appearing in the chosen equation. In the uniquely determined case H was one greater than D. In the indeterminate case H was two greater than D. And in the present overdetermined case H is equal to D. It looks as though it ought to be possible to set up rules for recognizing the three types of cases on the basis of the relation between H and D. As we shall see later, this statement is not entirely correct. However, it is true than an equation is likely to be uniquely determined if H is one greater than D, indeterminate if H is more than one greater than D, and overdetermined if H is equal to or less than D.

A GENERAL RULE FOR UNIQUENESS

It will be convenient to discuss the general uniqueness rule in terms of a notation more general than the one we have been using for the two-equation models. Let us assume that the chosen equation can be written

$$(28)\quad \beta_1 y_1 + \ldots + \beta_H y_H + \beta_{H+1}\, y_{H+1} + \ldots + \beta_G y_G + \gamma_1 z_1 + \ldots + \gamma_F z_F + \gamma_{F+1}\, z_{F+1} + \ldots + \gamma_{F+D}\, z_{F+D} = \zeta.$$

The y's are jointly determined variables and the z's are predetermined variables. There are G jointly determined variables and $F + D$ predetermined variables in the model. Exactly $G - H$ of the β's have the value zero, and exactly D of the γ's have the value zero. Usually it will be convenient to say that there are H jointly determined variables and F predetermined variables in the chosen equation; but this statement is to be taken to mean that there are H non-zero β's and F non-zero γ's. ζ is a disturbance term.

Let us divide the y's into two groups: The variables $x_1, x_2 \ldots x_H$ are the jointly determined variables that appear in the chosen equation and the variables $r_1, r_2 \ldots r_{G-H}$ are the jointly determined variables that appear in the model but do not appear in the chosen equation. Let us also divide the z's into two groups: The variables $u_1, u_2 \ldots u_F$ are the predetermined variables that appear in the chosen equation, and the variables $v_1, v_2 \ldots v_D$ are the predetermined variables that appear in the model but do not appear in the chosen equation.

At this point it is necessary to introduce certain ideas concerning matrices. Using matrix notation enables us to write, in one line, statements that would take several lines in ordinary algebraic notation. A matrix is a rectangular array of detached numbers. Thus

$$(29)\quad A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

is a matrix. Since there are 2 rows and 3 columns, it is a "2 x 3" matrix. Each of the a's is an element of the matrix; the element a_{ij} is the element in the i^{th} row and the j^{th} column. A matrix having just one row is a row matrix, and a matrix having just one column is a column matrix.

The transpose of a matrix is formed by making the i^{th} row the i^{th} column, and vice versa. Thus the transpose of A is

$$(30) \quad A' = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \end{bmatrix}.$$

We shall have occasion to multiply one matrix by another. Matrix multiplication differs from algebraic multiplication in that the order in matrix multiplication is important. Also there are matrices that cannot be multiplied. We can find the product AB (where A and B are matrices) if and only if the number of columns in A is equal to the number of rows in B. The product matrix C will then have as many columns as B and as many rows as A. The c_{ij} element of C is the sum of a number of products; each product is the product of an element of the i^{th} row of A and the corresponding element of the j^{th} column of B. For instance,

$$(31) \quad \overset{\textstyle A}{\begin{bmatrix} a_{11} & a_{12} \end{bmatrix}} \overset{\textstyle B}{\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}} =$$

$$\overset{\textstyle C}{\begin{bmatrix} a_{11} b_{11} + a_{12} b_{21} & a_{11} b_{12} + a_{12} b_{22} & a_{11} b_{13} + a_{12} b_{23} \end{bmatrix}}.$$

We can write our variables in matrix form. Thus the matrix of the jointly determined variables appearing in the chosen equation is the row matrix

$$(32) \quad x = [x_1 x_2 \ldots x_H].$$

The matrix of jointly determined variables appearing in the model but not appearing in the chosen equation is

$$(33) \quad r = [r_1 r_2 \ldots r_{G-H}].$$

The matrix of predetermined variables appearing in the chosen equation is

$$(34) \quad u = [u_1 u_2 \ldots u_F].$$

The matrix of predetermined variables appearing in the model but not appearing in the chosen equation is

$$(35) \quad v = [v_1 v_2 \ldots v_D].$$

In matrix form the chosen basic equation can be written as

$$(36) \quad \beta x' + \gamma u' = \zeta,$$

where β is a row matrix of the coefficients of the jointly determined variables and γ is a row matrix of the coefficients of the predetermined variables.

The isolated form equations for the x's can be written

$$(37)\quad x_1 = \pi^*_{x_1u_1} u_1 + \pi^*_{x_1u_2} u_2 + \dots + \pi^*_{x_1u_F} u_F + \pi_{x_1v_1} v_1 + \pi_{x_1v_2} v_2 + \dots + \pi_{x_1v_D} v_D + \delta_1$$

$$x_2 = \pi^*_{x_2u_1} u_1 + \pi^*_{x_2u_2} u_2 + \dots + \pi^*_{x_2u_F} u_F + \pi_{x_2v_1} v_1 + \pi_{x_2v_2} v_2 + \dots + \pi_{x_2v_D} v_D + \delta_2$$

. . . .

$$x_H = \pi^*_{x_Hu_1} u_1 + \pi^*_{x_Hu_2} u_2 + \dots + \pi^*_{x_Hu_F} u_F + \pi_{x_Hv_1} v_1 + \pi_{x_Hv_2} v_2 + \dots + \pi_{x_Hv_D} v_D + \delta_H .$$ [10]

But on the basis of what we have noticed about matrices we can condense these H equations into one matrix equation:

$$(38)\quad x' = \pi^*_{xu} u' + \pi_{xv} v' + \delta'.$$

x', u', v' and δ' are column matrices. π^*_{xu} is an H row and F column matrix, and π_{xv} is an H row and D column matrix. Direct multiplication will show that (38) includes all the terms in the equations (37).

Next let us multiply (38), term by term, by the matrix β (the row matrix of the coefficients of the jointly determined variables in the chosen behavior equation). The result is

$$(39)\quad \beta x' = \beta\pi^*_{xu} u' + \beta\pi_{xv} v' + \beta\delta'.$$

This equation is to be compared with (36). It is apparent that

$$(40)\quad \gamma = -\beta\pi^*_{xu} \qquad \text{and}$$

$$(41)\quad 0 = -\beta\pi_{xv}.$$

Once we have solved for the elements of β, (40) will enable us to solve for the elements of γ. We hope to use (41) to solve for the elements of β; the characteristics of π_{xv} determine whether we can do so.

[10]We must make the very mild assumption that at least one of the coefficients is different from zero.

We can rewrite (41) in the explicit form

$$(42)\qquad \begin{aligned} \beta_1\,\pi_{x_1v_1} + \beta_2\,\pi_{x_2v_1} + \cdots + \beta_H\,\pi_{x_Hv_1} &= 0 \\ \beta_1\,\pi_{x_1v_2} + \beta_2\,\pi_{x_2v_2} + \cdots + \beta_H\,\pi_{x_Hv_2} &= 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots &\\ \beta_1\,\pi_{x_1v_D} + \beta_2\,\pi_{x_2v_D} + \cdots + \beta_H\,\pi_{x_Hv_D} &= 0. \end{aligned}$$

(42) is a set of D simultaneous linear equations in the β's. Since the right-hand terms are all zeros, we can say that (42) is a set of homogeneous linear equations in the β's.

To see whether there are any useful solutions for the β's, it is convenient to introduce the idea of the determinant of a square matrix. Let us start with the matrix

$$(43)\qquad A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}.$$

The determinant of A (written $|A|$) is the sum of all the possible products that can be formed in the following way:

1. Each product must consist of as many factors (elements) as there are rows in the matrix.
2. One factor must come from each row.
3. One factor must come from each column.
4. Each product has a plus sign if an even number of interchanges of adjacent second subscripts would make the order of the second subscripts the same as the order of the first subscripts, and a minus sign if an odd number of interchanges of adjacent second subscripts would make the two orders the same.

The determinant of A contains the following terms:

1.	$a_{11}\ a_{22}\ a_{33}$	(+)
2.	$a_{11}\ a_{32}\ a_{23}$	(—)
3.	$a_{21}\ a_{13}\ a_{33}$	(—)
4.	$a_{21}\ a_{32}\ a_{13}$	(+)
5.	$a_{31}\ a_{12}\ a_{23}$	(+)
6.	$a_{31}\ a_{22}\ a_{13}$	(—).

Let us check the sign of the 4^{th} term. The order of the second subscripts is 123. It we interchange the 1 and the 2, and then interchange the 1 and the 3, we have 231, the order of the first subscripts. Since we needed two interchanges of adjacent second subscripts, the 4^{th} term has a positive sign.

For the time being let us assume that D is equal to H, so that (42) is a set of H simultaneous equations in the H unknown β's. Call the determinant of the coefficients of the

β's $\triangle$. Then the usual procedure in solving for the i^{th} element of β is that of setting β_i equal to a ratio of two dedeterminants. The denominator is $\triangle$. The numerator is $\triangle$ with the i^{th} column of coefficients (the coefficients of β_i) replaced by the column of constant terms appearing on the right sides of the simultaneous equations.

Every one of the numerators is zero, since each of them is a determinant having a column of zeroes. Since this column must be represented in each term of the determinant, the determinant is the sum of a collection of terms, each of which is zero.

If the denominator ($\triangle$) is not zero, then every β element is zero; in this case we have what is called a trivial solution which is of no use. Therefore $\triangle$ must be zero if we are to proceed in finding useful values for the elements of β. In matrix terminology the *rank* of the π_{xv} matrix must be less than H. (The rank of a matrix is R if and only if every determinant formed by taking more than R of its rows and more than R of its columns is zero, while at least one determinant formed from R of its rows and R of its columns is not zero.)

If the rank of the matrix is $H - 1$, then there are just $H - 1$ (instead of H) independent equations in the elements of β. But if we are willing to assign a value to one of the β elements, we will be left with $H - 1$ equations in $H - 1$ unknown elements of β. Fortunately, we do not lose anything by assigning a value to one of the elements of β; doing so simply amounts to dividing the whole chosen basic equation by a constant.

If the rank of the π_{xv} matrix is less than $H - 1$, we have less than $H - 1$ independent equations; and after we assign a value to one of the elements of β we have still $H - 1$ unknown elements of β. Plainly we cannot expect to be able to solve for the β elements.

Thus we can solve for the elements of β if and only if the rank of the π_{xv} matrix is $H - 1$. If the rank is more than $H - 1$ we have too many independent equations; the only consistent solution is the trivial one in which every element of β is zero. If the rank is less than $H - 1$ the β matrix is indeterminate—an infinite number of combinations of elements of β will satisfy the isolated form equations.

Temporarily we have been assuming that D is equal to H. But if we remove this assumption the rank requirement is unchanged. However, it is clear that the rank of π_{xv} cannot be as large as $H - 1$ if D is less than $H - 1$.

Let us check our two-equation models to see whether our present rule is consistent with the results obtained. In the uniquely determined case in equation (10), H is 2 and D is 1. The isolated form equations are (12) and (13). Since p and q appear in (10) and I does not, the π_{xv} matrix is

$$(44) \quad \begin{bmatrix} \dfrac{-\delta}{1-\alpha\eta} \\[2ex] \dfrac{\alpha\delta}{1-\alpha\eta} \end{bmatrix}.$$

The rank of this matrix is 1, if we assume that neither α nor δ is zero. Our rule predicts correctly that we can solve for α.

In the indeterminate case (equation (11)), H is 2 and D is zero. Of course the matrix π_{xv} consists of zeroes, and so has the rank zero. As we expect, this is the case in which an infinite number of values of γ are possible; the solution for γ is indeterminate.

We found that equation (18) was overdetermined. H was 2 and D was 2. The isolated form equations (20) and (21) make it look as though the rank of the π_{xv} matrix

$$(45) \quad \begin{bmatrix} \dfrac{-\delta}{1-\alpha\eta} & \dfrac{-\epsilon}{1-\alpha\eta} \\[2ex] \dfrac{\alpha\delta}{1-\alpha\eta} & \dfrac{\alpha\epsilon}{1-\alpha\eta} \end{bmatrix}$$

were 1. But we do not know the elements of this matrix; we must estimate them by least squares. When we substitute into (45) the least squares estimates from equations (22) through (25), matrix (45) takes the form

$$(46) \quad \begin{bmatrix} \dfrac{\begin{vmatrix} \Sigma qI & \Sigma IC \\ \Sigma qC & \Sigma C^2 \end{vmatrix}}{\triangle} & \dfrac{\begin{vmatrix} \Sigma I^2 & \Sigma qI \\ \Sigma IC & \Sigma qC \end{vmatrix}}{\triangle} \\[4ex] \dfrac{\begin{vmatrix} \Sigma pI & \Sigma IC \\ \Sigma pC & \Sigma C^2 \end{vmatrix}}{\triangle} & \dfrac{\begin{vmatrix} \Sigma I^2 & \Sigma pI \\ \Sigma IC & \Sigma pC \end{vmatrix}}{\triangle} \end{bmatrix}, \qquad \text{where}$$

$$(47) \quad \triangle = \begin{bmatrix} \Sigma I & \Sigma IC \\ \Sigma IC & \Sigma C^2 \end{bmatrix}.$$

Direct multiplication shows that (46) has the rank 2. And this is the case where we have two inconsistent estimates of α.

Strickly speaking the rule must be stated in terms of the rank of π_{xv}. But usually we can predict uniqueness on the basis of the relation between H and D. If D is equal to H — 1, we can be fairly confident that the chosen behavior equation is uniquely determined, although it may be indeterminate. If D is at least as large as H then the equation must be overdetermined, and if D is less than H — 1 the equation must be indeterminate.

MAXIMUM LIKELIHOOD

The maximum likelihood procedure is a method of arriving at a compromise among estimates. In the uniquely determined case there is no need for compromise, of course. In the indeterminate case, compromise would be meaningless since there is an infinite number of possible combinations of values of the behavior coefficients. But in the overdetermined case we have a finite number of estimates, and a compromise is conceivable.[11]

Let us return to the isolated form equations (20) and (21). We can rewrite them as

$$(48) \quad p = \pi_{11} I + \pi_{12} C + \epsilon_1 \quad \text{and}$$

$$(49) \quad q = \pi_{21} I + \pi_{22} C + \epsilon_2.$$

We assume that the residuals ϵ_1 and ϵ_2 are jointly normally and independently distributed. We wish to choose the π's so that the joint probability of the residuals will be maximized. The method of doing so is the maximum likelihood method.[12]

There are TH residuals, since we have T observations on each of the H jointly determined variables. The joint probability of getting a particular collection of residuals depends on the isolated form coefficients and the elements of the *covariance matrix* of the residuals. The ij element of the covariance matrix is

$$(50) \quad \sigma_{ij} = \frac{\Sigma_{i,j} \, (\text{residual}_i \times \text{residual}_j)}{T}.$$

We wish to choose the isolated form coefficients and the elements of the covariance matrix of the residuals in such a way that the probability density of the set of TH residuals that we actually have will be maximized.

[11]A compromise involves using only a part of the available information; accordingly the Anderson-Rubin procedure to be described is called the "limited information" method.

[12]See A. M. Mood, Introduction to the theory of statistics. McGraw-Hill, N. Y. 1950. pp. 152, ff.

The maximization process must be carried on subject to two side conditions. The first is that the equations (41), pertaining to the relation between the β coefficients and the π coefficients, must be satisfied. The second condition is called a *normalization condition.* In its most general form it is

$$(51) \quad \beta\Phi_{xx}\beta' = 1,$$

where Φ_{xx} can be either a known constant or a known function of unknown parameters. In our case, (51) takes the form

$$(52) \quad \beta_1 = 1,$$

since in (18) the coefficient of p has been chosen as 1.[13]

Lagrange multipliers are used to take account of the side conditions (41) and (51).[14] Thus the maximization process includes maximization of the residuals' joint probability with respect to the elements of β, as well as the elements of π and the covariance matrix of the residuals. The function to be maximized is thus the likelihood function plus two terms involving Lagrange multipliers. We set equal to zero the derivative of the augmented function with respect to each element of the β, π and covariance matrices. When we solve the simultaneous equations we have the maximum likelihood estimates of the elements of these matrices. For the chosen behavior equation, the coefficients of the predetermined variables can be estimated from the π coefficients appearing in the isolated form equations. The details of the algebra will not be presented here.

Let us restate the nature of this latest step in our procedure: In the overdetermined case there were too many indications of the values of the isolated form coefficients. Use of the maximum likelihood method is a way of com-

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[13]It is clear that no damage is done by adopting such a convention. We wish to know how a change in one variable affects another variable; therefore only the relative size of the coefficients is significant.

[14]An example will make the nature of a Lagrange multiplier clear enough for the present purpose. Assume that we wish to maximize the area of a rectangle, subject to the side condition that the length (x) plus the breadth (y) must be 10. We maximize xy subject to $x + y - 10 = 0$. We can maximize $xy - 0$, or $xy + \lambda 0$, where λ is any finite number. Since $x + y - 10 = 0$, we can maximize $xy + \lambda(x + y - 10)$. We have already put the side condition into the expression to be maximized, so we can maximize with respect to x and y independently. We set equal to zero the derivative of the quantity to be maximized with respect to x and y. Thus we have $y + \lambda = 0$, and $x + \lambda = 0$. These equations plus the equation $x + y - 10 = 0$ are three simultaneous equations that we can solve for x, y, and λ. In this case we are not finally interested in the value of λ. We find $x = y = 5$; these dimensions will give us our rectangle of maximum area, of course.

promising the claims of the various indications. There are many ways in which the compromise could have been made. The particular compromise associated with maximum likelihood is the one which gives us estimates of the relevant coefficients maximizing the probability of finding the set of isolated form residuals that we have actually observed.

THE LIKELIHOOD RATIO TEST FOR OVERDETERMINATION

For convenience in computation the matrix v is replaced by the matrix s, which consists of the part of v that is orthogonal to u.[15] So we shall refer to the π_{xs} matrix instead of the π_{xv} matrix.

Let P_{xs} denote the matrix of estimates of the elements of π_{xs}. P_{xs} is almost certain to have the rank H rather than $H-1$. But if the determinant of P_{xs} is nearly zero we need not reject the hypothesis that the sample comes from a population in which the π_{xs} matrix has the rank $H-1$.

To decide whether to proceed on the basis of the estimates, we can use the test statistic

$$(53)\quad V = T \log (1 + v).$$

T is the number of observations on each variable. v depends on the estimates of the elements of π_{xs} and on the estimates made by the maximum likelihood method. v is relatively low if the P_{xs} matrix is relatively near to having the rank $H-1$ (instead of the rank H). The test statistic (53) is formed by applying the likelihood ratio principle, so it is distributed asymptotically as χ^2, with $D-H+1$ degrees of freedom.[16] If V is less than the critical χ^2 value, then we will proceed to estimate the chosen behavior equation.[17]

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[15]That is, if we multiply s_t by u_t, term by term, the sum of the products will be zero.

[16]That is, the distribution of V approaches the distribution of χ^2 as the number of observations approaches infinity. On the subject of the likelihood ratio test, see A. M. Mood, Introduction to the theory of statistics. McGraw-Hill, New York. 1950. pp. 257, ff.

[17]The validity of this test depends on the fulfillment of a large number of conditions. Some of them are as follows: The covariance matrix limit of the predetermined variables must converge to a fixed non-singular limit in probability; the residuals in the isolated form equations must tend in probability to be independent of the predetermined variables; in each of the behavior equations the disturbance term must have a mean of zero, and be serially independent. For the complete list of assumptions, see T. W. Anderson and Herman Rubin, The asymptotic properties of estimates of the parameters of a single equation in a complete system of stochastic equations, 21 Annals of Mathematical Statistics 571 ff., December 1950.

TESTING FOR RANDOMNESS OF RESIDUALS[18]

In dealing with the isolated form equations we have used the least squares procedure. This procedure assumes that residuals measured from the regression line are random around the regression line. There must be no autocorrelation among the residuals. The fact that the residual at time t is positive must give us no reason to expect that the residual at time $t+1$ will be positive, and no reason to expect that it will be negative.

But even if there is no autocorrelation among residuals in the population, autocorrelation may appear in the sample of T observations. We wish to decide in a particular case whether the sample autocorrelation is high enough to force us to conclude that there must be autocorrelation in the population.

The test to be used is one devised by von Neumann. Strictly, the test is a test of autocorrelation among disturbances. But since the disturbances are not observed, in practice the test is used on residuals. This substitution reduces the usefulness of the test, of course. The sample autocorrelation among residuals is evaluated in relation to the variance of estimate. Define a statistic R such that

$$(54) \quad R = \frac{\Delta^2}{S^2} \qquad \text{where}$$

$$(55) \quad \Delta^2 = \frac{\sum_{1}^{T-1} (d_t - d_{t-1})^2}{T-1} \qquad \text{and}$$

$$(56) \quad S^2 = \frac{\sum_{1}^{T} d_t^2}{T},$$

d_t being the residual at time t. Δ^2 is a measure of autocorrelation.

If the population residuals are random, the most likely value for R is $2T/T-1$. The distribution of R has been tabulated for various levels of confidence.[19] If the auto-

[18]Testing for randomness of residuals is not part of the procedure used by Anderson and Rubin.

[19]B. S. Hart and J. von Neumann, Tabulation of the probabilities for the ratio of the mean square successive difference to the variance, 13 Annals of Mathematical Statistics 207, June 1942.

correlation is strongly positive, R will be less than the lower critical value associated with a given level of probability; if the correlation is strongly negative, R will be greater than the higher critical value associated with the given level of probability.

In the overdetermined case we used the maximum likelihood method. This method is based partly on the assumption that the residuals at different times are jointly normally and independently distributed. If this assumption is to be realized, there must be no autocorrelation among the residuals. The von Neumann test ratio can be used on the residuals from the chosen behavior equation, although the test is strictly valid only for disturbances, rather than residuals.

CONFIDENCE INTERVALS FOR β AND γ

Under certain assumptions it is possible to set up confidence intervals for β and γ.[20] Since this has not been done in the present study, the process will not be described.

APPLICATION OF METHODS

PORK, BEEF AND POULTRY PRODUCTS

SIMULTANEOUS EQUATIONS METHOD

OVERDETERMINED MODEL

The variables used are logarithms and cover the period 1921 through 1941. The jointly determined variables are logarithms of index numbers. Where no deflating is involved, the index numbers have 1935-39 as their base period. Where index numbers are formed by deflating a price series by the consumer price index, both the individual price series and the consumer price index series are based on 1935-39. Jointly determined variables will be denoted by y; predetermined variables by z.

The index numbers are index numbers of:

y The logarithm of:

1. Per capita quantity of pork sold at retail (quantity of pork, U. S. Bureau of Agricultural Economics, Consumption of food in the United States 1909-48, Miscellaneous publication #691, Aug. 1949, p. 109; Population of the continental United States, U. S. Bureau of the Census, Statistical Abstract of the United States, 1950, p. 8).
2. Retail price of pork deflated by the consumer price index (price of pork, U. S. Bureau of Agricultural Economics, Price spreads between farmers and consumers, Agricultural information bulletin

[20]See footnote 17, page 1001.

#4, 1949; consumer price index, U. S. Bureau of Labor Statistics, Handbook of labor statistics, 1947 edition, Bulletin #916, p. 107).

3. Retail price of beef deflated by consumer price index (retail price of beef, U. S. Bureau of Agricultural Economics, The marketing and transportation situation, data appearing monthly in tables showing price spreads between farmers and consumers).
4. Retail price of poultry products deflated by consumer price index (retail price of poultry products, U. S. Department of Agriculture, Agricultural statistics: (a) an index of prices received by farmers, p. 620 and (b) data on the farmers' share of consumers' dollars, p. 621).
5. Retail price of dairy products deflated by consumer price index (retail price of dairy products, U. S. Bureau of Labor Statistics, Handbook of labor statistics, 1947 edition, Bulletin #916, p. 121).
6. Retail price of oleomargarine deflated by consumer price index (retail price of oleomargarine, U. S. Bureau of Agricultural Economics, Agricultural outlook charts, 1950, Oct. 1949, p. 53).
7. Per capita quantity of beef sold at retail (quantity of beef, U. S. Bureau of Agricultural Economics, Consumption of food in the United States 1909-48, Miscellaneous publication #691, Aug. 1949, p. 109).
8. Per capita quantity of poultry products sold at retail (ibid., p. 111).
9. Per capita quantity of dairy products sold at retail (ibid., p. 73).
10. Per capita quantity of oleomargarine sold at retail (ibid., p. 113).
11. Per capita feed grain disappearance (U. S. Bureau of Agricultural Economics, Feed statistics, Statistics bulletin #85, Dec. 1949, p. 27).
12. Price of feed grains deflated by consumer price index (price of feed grains, ibid., p. 40).
13. Per capita quantity of feed grains produced (U. S. Bureau of Agricultural Economics, Feed statistics, March 1949, p. 6).

The predetermined variables are also logarithms of index numbers, with the same conventions just enumerated for the jointly dependent variables. The index numbers are index numbers of:

z *The logarithm of*:

1. 10 raised to an exponent equal to the number of years by which the given year is removed from the year 1921.
2. 10 raised to an exponent equal to the square of the number of years by which the given year is removed from 1921.
3. Retail price of foods other than meats, poultry products, dairy products and oleomargarine, deflated by consumer price index (retail price of food, U. S. Bureau of Labor Statistics, Bulletin #916, 1947 edition, p. 121. Weights of foods excluded are indicated in this bulletin).
4. Per capita disposable income deflated by consumer price index (disposable personal income after 1928, U. S. Department of Commerce, National income supplement to the survey of current business, July 1947, p. 19; 1928 and earlier years, U. S. Bureau of Agricultural Economics, Outlook charts 1948, p. 10, estimates based on Department of Commerce data).
5. Weighted average of preceding 5 years' values for z_4 (weights: 10 for $z_{4,t-1}$, 4 for $z_{4,t-2}$, 3 for $z_{4,t-3}$, 2 for $z_{4,t-4}$ and 1 for $z_{4,t-5}$).
6. Pasture conditions August 1 lagged one year (U. S. Bureau of Agricultural Statistics, Feed statistics, Statistical bulletin #85, Dec. 1949, p. 84).

7. Price of fats used in making oleomargarine deflated by consumer price index (prices of oleo oil, neutral lard and coconut oil, U. S. Bureau of Agricultural Economics, Fats and oil situation, Jan.-Feb. 1946, pp. 23, 24, 26; prices of cottonseed oil and soybean oil, unpublished data compiled by the Bureau of Agricultural Economics).
8. Feed grain yield per acre (U. S. Bureau of Agricultural Economics, Feed statistics, Statistical bulletin #85, Dec. 1949, p. 5).
9. One-year-lagged value of y_{12}.
10. One-year-lagged value of z_8.
11. Per capita feed grain carry-over (feed grain carry-over, 1926 and later years, U. S. Bureau of Agricultural Economics, Feed statistics, Statistical bulletin #85, Dec. 1949, pp. 16-18; before 1926, ibid., Feb. 1940, supplement).

All the data are yearly figures; data for shorter periods are not available regularly enough to make their use feasible. Using annual data means a considerable loss of information. In effect we are using annual averages; variations during the year may be highly significant for an understanding of demand functions, but we are prevented from seeing these variations.[21]

The weights for z_5 are chosen roughly on the assumption that the influence of past income of present spending diminishes as the past income recedes into the distance. In the present stage of knowledge about consumer behavior the specific choices are partly arbitrary.

This model is said to be overdetermined because the demand equations for pork, beef and poultry products (the chosen equations) are overdetermined. In each of these equations, H (the number of jointly determined variables appearing in the chosen equation) is 6 and D (the number of predetermined variables appearing in the model but not appearing in the chosen equation) is also 6. The fact that D is more than H — 1 makes each chosen equation overdetermined.

The overdetermined model includes one equation for each of the principal types of decision. There are demand equations for pork, beef and poultry products. There are demand equations that help to determine the prices of goods that compete with these products for the use of farmers' resources. There is a demand equation for feed grains, whose price enters into farmers' decisions on the scale of their livestock enterprises. There is a supply equation for each good for which there is a demand equation. In addition there is a production equation for feed grains, since production helps to determine carry-over, and carry-over helps to determine the scale of feeding.

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[21]See Sten Malmquist, A statistical analysis of the demand for liquor in Sweden, Uppsala, Appelbergs Boktryckeriaktiebolaget. 1948. pp. 81, ff.

All the equations in the overdetermined model are linear in the logarithms of the variables. Unfortunately, it is not practicable to include several types of equations in the same model; for one thing, computation cost would be prohibitive. Making all the equations non-linear would be possible, but would be very expensive with a relatively large number of variables. While it would be possible to make each equation linear in any functions of the variables, probably the choice is likely to be a choice between making the equations linear in the variables and making them linear in the logarithms of the variables.

The former choice involves the assumption that within each equation the partial effect of any variable on any other variable is invariant as we vary the values of all the remaining variables in the equation. For instance, if we made each equation linear in the variables we would be assuming that the quantity effect of any price change is independent of the level of income. Such assumptions are likely to be damagingly restrictive. The assumptions involved in making the equations linear in the logarithms appear to be less inconvenient. For instance, the demand curve is assumed not to touch either axis. But within the range of prices that is relevant to significant price problems this assumption has no effect.

The specific form of the demand equations is not derived from a general analysis of indifference function. Even a simple indifference function is likely to lead to a rather complicated demand function. For instance, suppose we have only two goods, q_1 and q_2, and the amount of money to be spent on both of them is E. Let the prices be given as p_1 and p_2. Assume that the indifference function is a general second degree equation

$$(57) \quad u = aq_1 + bq_1^2 + cq_1 q_2 + dq_2 + eq_2^2,$$

where u is the indifference index, considered determinate up to constants of addition and multiplication. The budget equation is

$$(58) \quad E = q_1 p_1 + q_2 p_2.$$

Maximize u by using a Lagrange multiplier λ. That is, find a stationary point for

$$(59) \quad G = u + \lambda (E - q_1p_1 - q_2p_2).$$

We have

$$(60) \frac{\partial G}{\partial q_1} = 0 = a + 2bq_1 + cq_2 + \lambda p_1 \qquad \text{and}$$

$$(61)\ \frac{\partial G}{\partial q_2} = 0 = cq_1 + d + 2eq_2 + \lambda p_2.$$

Solving (58), (60) and (61) for q_2, we have the demand equation

$$(62)\quad q_2 = \frac{E(cp_1 - 2bp_2) - p_1(ap_2 - dp_1)}{p_2(cp_1 - 2bp_2) + p_1(cp_2 - 2ep_1)}.^{22}$$

The demand equation for q_1 is of the same type as (62). Such equations would be highly inconvenient to work with, of course.

Moreover, there is no reason to think that making our rough approximations of functions at the indifference curve level is better than making them at the demand curve level. If the indifference map is conceptually observable through our asking hypothetical questions, then so is the demand curve.

The demand equations of the overdetermined model are:

(63) Pork

$$y_1 + \beta_{12}y_2 + \beta_{13}y_3 + \beta_{14}y_4 + \beta_{15}y_5 + \beta_{16}y_6$$
$$+ \gamma_{11}z_1 + \gamma_{12}z_2 + \gamma_{13}z_3 + \gamma_{14}z_4 + \gamma_{15}z_5 = \zeta_1$$

(64) Beef

$$\beta_{22}y_2 + \beta_{23}y_3 + \beta_{24}y_4 + \beta_{25}y_5 + \beta_{26}y_6 + y_7$$
$$+ \gamma_{21}z_1 + \gamma_{22}z_2 + \gamma_{23}z_3 + \gamma_{24}z_4 + \gamma_{25}z_5 = \zeta_2$$

(65) Poultry products

$$\beta_{33}y_3 + \beta_{34}y_4 + \beta_{35}y_5 + \beta_{36}y_6 + y_8 + \gamma_{31}z_1$$
$$+ \gamma_{32}z_2 + \gamma_{33}z_3 + \gamma_{34}z_4 + \gamma_{35}z_5 = \zeta_3$$

(66) Dairy products

$$\beta_{42}y_2 + \beta_{43}y_3 + \beta_{44}y_4 + \beta_{45}y_5 + \beta_{46}y_6 + y_9$$
$$+ \gamma_{41}z_1 + \gamma_{42}z_2 + \gamma_{43}z_3 + \gamma_{44}z_4 + \gamma_{45}z_5 = \zeta_4$$

(67) Oleomargarine

$$\beta_{52}y_2 + \beta_{53}y_3 + \beta_{54}y_4 + \beta_{55}y_5 + \beta_{56}y_6 + y_{10}$$
$$+ \gamma_{51}z_1 + \gamma_{52}z_2 + \gamma_{53}z_3 + \gamma_{54}z_4 + \gamma_{55}z_5 = \zeta_5$$

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[22]Since (58), (60) and (61) are linear equations in q_1, q_2 and λ, there is just one solution. This solution will be a maximum, if some rather mild assumptions about the coefficients in (57) are fulfilled.

(68) Feed grains

$$\beta_{62}y_2 + \beta_{63}y_3 + \beta_{65}y_5 + \beta_{66}y_6 + y_{11} + \beta_{6,12}y_{12} + \gamma_{61}z_1 + \gamma_{62}z_2 = \zeta_6.$$

The supply equations of the overdetermined model are:

(69) Pork

$$y_1 + \beta_{72}y_2 + \beta_{73}y_3 + \beta_{74}y_4 + \beta_{75}y_5 + \beta_{76}y_6 + \beta_{7,12}y_{12} + \gamma_{71}z_1 + \gamma_{72}z_2 + \gamma_{7,12}z_{12} = \zeta_7$$

(70) Beef

$$\beta_{82}y_2 + \beta_{83}y_3 + \beta_{84}y_4 + \beta_{85}y_5 + \beta_{86}y_6 + y_7 + \beta_{8,12}y_{12} + \gamma_{81}z_1 + \gamma_{82}z_2 + \gamma_{86}z_6 + \gamma_{8,12}z_{12} = \zeta_8$$

(71) Poultry products

$$\beta_{92}y_2 + \beta_{93}y_3 + \beta_{94}y_4 + \beta_{95}y_5 + \beta_{96}y_6 + y_8 + \beta_{9,12}y_{12} + \gamma_{91}z_1 + \gamma_{92}z_2 + \gamma_{9,11}z_{11} = \zeta_9$$

(72) Dairy products

$$\beta_{10,2}y_2 + \beta_{10,3}y_3 + \beta_{10,4}y_4 + \beta_{10,5}y_5 + \beta_{10,6}y_6 + y_9 + \beta_{10,12}y_{12} + \gamma_{10,1}z_1 + \gamma_{10,2}z_2 + \gamma_{10,6}z_6 + \gamma_{10,11}z_{11} = \zeta_{10}$$

(73) Oleomargarine

$$y_6 + \beta_{11,10}y_{10} + \gamma_{11,1}z_1 + \gamma_{11,2}z_2 + \gamma_{11,7}z_7 = \zeta_{11}$$

(74) Feed grains

$$y_{11} + \beta_{12,12}y_{12} + \gamma_{12,1}z_1 + \gamma_{12,2}z_2 + \gamma_{12,9}z_9 + \gamma_{12,11}z_{11} = \zeta_{12}.$$

The one production equation of the overdetermined model is:

(75) Feed grains

$$y_{13} + \gamma_{13,1}z_1 + \gamma_{13,2}z_2 + \gamma_{13,8}z_8 + \gamma_{13,9}z_9 + \gamma_{13,10}z_{10} = \zeta_{13}.$$

Equation (63) relates per capita pork consumption to the following jointly determined variables: The prices of pork, beef, poultry products, dairy products and oleomargarine. All these prices are deflated by the consumer price index. Deflating is one way of taking account of the influence of prices of commodities other than those enumerated. It is a rough way, since the prices of the other commodities do not enter individually. Moreover, deflating involves the assumption that a proportionate change in all prices would leave all quantities unchanged; probably the influence of changes in the real values of stocks of money would make this assumption less than perfectly realistic.

Equation (63) also relates per capita pork consumption to time; the price index of foods other than meats, poultry products, dairy products and oleomargarine; and current and lagged disposable income. Introducing time explicitly reduces the danger of getting spurious correlations among the other variables. Also time trends presumably are associated with changes in tastes and customs over time. If time appears to have a strong influence on the results of the analysis, then further investigation may properly be directed at finding more selective series representing the social changes that have taken place over the relevant period.

Perhaps some difficulty is introduced when time is used as one of the exogenous variables. If the size of the sample approached infinity, t would approach infinity, and the M_{zz} matrix would not remain finite. However, introducing time as a variable is equivalent to stating the values of the other variables in terms of deviations from a time trend.[23] This operation appears to be a very reasonable one, despite the difficulty in the limit.[24]

Time is represented by both a first degree term and a second degree term, so that the time trend need not be linear. However, some danger is introduced in this connection since the two time terms will be correlated.

The interpretations of equations (64), (65), (66) and (67) are similar to the interpretation of (63). Equations (63), (64) and (65) are the ones whose coefficients are to be estimated. It is assumed that β_{12}, β_{23} and β_{34} are all negative—that an increase in the price of any of the three kinds

[23]This was proved for linear trends by Frisch and Waugh. It was generalized to non-linear trends by Tintner. See Gerhard Tintner, Econometrics. Wiley, New York. 1952. pp. 301, ff.

[24]The procedure is usual in econometric work. See, for example, Girshick and Haavelmo. Statistical analysis of the demand for food. 15 Econometrica 79. April 1947.

of meat will be accompanied by a reduction in the consumption of that same kind of meat. Since none of these goods appears to be an inferior good, it is also assumed that the signs of γ_{14}, γ_{24} and γ_{34} are all positive. That is, it is assumed that high current income is associated with consumption of large quantities of meat products. A model giving estimates inconsistent with these assumptions will be considered suspect.

The first four supply equations deal with the decisions of farmers who must decide how much emphasis to put on the production of the meat in question, and how much on dairy production.

The quantity of dairy products produced may depend partly on the price of margarine; in any case the price of dairy products depends partly on the price of margarine, so there must be a demand equation for it. Of course, there must also be a demand equation for dairy products, since dairying is one of the enterprises competing with meat production. If the price of margarine is to be used, then it must also have a supply function.

There are three equations pertaining to feed grains. Farmers' decisions about production of feed grains depends on yield and acreage planted. Acreage planted is represented as depending on last year's price and yield of feed grains. If last year's yield was unusually large, and if the elasticity of feed grain demand is less than 1, perhaps the farmers will expect that this year the desirability of producing the feed will increase; yield may be expected to be more nearly "normal" (lower). Or perhaps the farmers will increase their acreage of feed crops in this situation in an attempt to recoup last year's losses by putting more of their land to a use that is relatively profitable in the short run. Disappearance is conceived of as the quantity sold for use in feeding livestock; in effect a farmer who uses homegrown feed grains is thought of as selling the grains to himself. The demand for feed grains depends on the prices of livestock products. The supply depends on the carry-over and on the current and lagged prices of feed grains. The lagged price is included because owners of feed grains may judge the desirability of the current price partly on the basis of its relation to last year's price.

All production stages between the farmer and the consumer are omitted from this model. The farmer is represented as reacting directly to the retail prices of products which are processed after leaving his farm. In effect, this amounts to assuming that the farmer reacts to a constant times each price—that the combined margins of processors and retailers are a constant percentage of the retail price. In practice this assumption may not be far from reality.

Although it would be desirable to base the supply equations specifically on some profit-maximizing model of farmers' behavior, the algebraic difficulties involved are too formidable. For instance, let us consider two of the simplest meaningful production situations. In both we assume that the farmer has a limited amount of funds, K, to be spent in hiring quantities of resources x and y during one production period. He has two enterprises, turning out products u and v. Let x_u be the quantity of x used in producing u, and let x_v, y_u and y_v be interpreted accordingly. Let the prices of the products be p_u and p_v and the rents of the resources, p_x and p_y. In the first case, assume that the production functions are

$$(76)\quad u = x_u^{\alpha} y_u^{\beta} \qquad \text{and}$$

$$(77)\quad v = x_v^{\gamma} y_v^{\delta}.$$

The farmer's budget equation is

$$(78)\quad p_x (x_u + x_v) + p_y (y_u + y_v) = K.$$

We want to get an equation which shows u, for instance, as a function of the prices p_u and p_v with K, p_x, p_y, α, β, γ and δ assumed constant. To do this we maximize the profit with respect to the four resource quantities. We solve the resulting equations for the four resource quantities. If this can be done conveniently, then it is easy to use the production functions to get the quantities of product as functions of the relevant prices. However, in the present case the equation to be solved for x_u is

$$(79)\quad K_1 p_u x_u^{\alpha+\beta-1} = K_2 p_v \left[\frac{K}{p_x} - x_u^{\frac{\alpha+\beta}{\alpha}}\right]^{\gamma+\delta-1},$$

where K_1 and K_2 are constants.

Plainly this equation is too complicated to be used conveniently in setting up simple equations for the supply functions of the various kinds of output.

Alternatively we may try the production functions

$$(80)\quad u = ax_u + bx_u^2 + cx_u y_u + dy_u + ey_u^2 \qquad \text{and}$$

$$(81)\quad v = fx_v + gx_v^2 + hx_v y_v + gy_v + ky_v^2.$$

In this case we are fortunate in having linear relations to work with after we take the derivatives of the profit function with respect to the quantities of resources. But the results are still inconvenient. For instance, the solution for x_u is

(82)

$$x_u = \frac{\begin{vmatrix} -ap_u & cp_u & o & o & p_x \\ -dp_u & 2ep_u & o & o & p_y \\ -fp_v & o & 2gp_v & hp_v & p_x \\ -gp_v & o & hp_v & 2kp_v & p_y \\ K & p_y & p_x & p_y & o \end{vmatrix}}{\begin{vmatrix} 2bp_u & cp_u & o & o & p_x \\ cp_u & 2ep_u & o & o & p_y \\ o & o & 2gp_v & hp_v & p_x \\ o & o & hp_v & 2kp_v & p_y \\ p_x & p_y & p_x & p_y & o \end{vmatrix}}.$$

In this case, too, the expression for x_u together with a similar expression for y_u would give us a much too complicated expression for u. The simple supply functions in the text above may be regarded as very rough approximations to the supply functions consistent with the economic theory that has been examined. Further study might suggest more convenient approximations.

When we solve each of the equations for the quantity of the meat in question, we have

(83) Pork

$$\begin{aligned} y_1 = & -0.81y_2 - 1.50y_3 - 1.88y_4 + 5.53y_5 \\ & + 1.72y_6 + 0.11z_1 - 0.31z_2 - 0.77z_3 \\ & - 0.40z_4 + 0.26z_5 - 2.98 \end{aligned}$$

(84) Beef

$$\begin{aligned} y_7 = & +0.72y_2 - 3.23y_3 - 2.50y_4 + 7.41y_5 \\ & + 1.86y_6 + 0.14z_1 - 0.34z_2 - 1.34z_3 \\ & - 0.63z_4 - 0.10z_5 - 3.35 \end{aligned}$$

(85) Poultry Products

$$\begin{aligned} y_8 = & \ 2.05y_3 + 2.28y_4 - 7.06y_5 - 2.51y_6 \\ & - 0.15z_1 + 0.32z_2 + 1.32z_3 + 1.59z_4 \\ & + 0.27z_5 + 6.59. \end{aligned}$$

The reason for the exclusion of the pork price from the poultry demand equation is as follows: In a preliminary attempt at solving the equations there was difficulty because the pork price was highly correlated with the other variables appearing in the poultry demand equation. Omitting it made the computation manageable.

In equations (83), (84) and (85), all the variables are in logarithmic form. Each equation is of the type

$$(86) \quad \log Y = a_1 \log X_1 + a_2 \log X_2 + \ldots + a_n \log X_n \text{ or}$$

$$(87) \quad Y = X_1^{a_1} X_2^{a_2} \ldots X_n^{a_n}.$$

Let us take the derivative of Y with respect to an X—say X_1:

$$(88) \quad \frac{\partial Y}{\partial X_1} = a_1 X_1^{a_1-1} X_2^{a_2} \ldots X_n^{a_n}.$$

The elasticity of Y with respect to X_1 is

$$(89) \quad \epsilon = \frac{\partial Y}{\partial X_1} \cdot \frac{X_1}{Y} = \frac{a_1 X_1^{a_1} X_2^{a_2} \ldots X_n^{a_n}}{X_1^{a_1} X_2^{a_2} \ldots X_n^{a_n}} = a_1.$$

Thus each of the coefficients in (86) is an elasticity. (83) states, for instance, that a 1 percent increase in the price of pork, income and other prices remaining unchanged, will lead to an 0.8 percent decrease in pork comsumption.

The residuals from both the isolated form equations and the basic equations are random on the basis of the von Neumann test. For 21 observations, the most likely value of R is 2.10(2n/n—1) for both reduced form and structural equations. In the reduced form equations R has the following values:

Independent variable	R	Independent variable	R
y_1	2.53	y_5	2.55
y_2	2.60	y_6	2.51
y_3	2.82	y_7	2.99
y_4	2.22	y_8	2.31

At the 5 percent level of significance, the critical values of R are 1.30 and 2.90; at the 1 percent level the critical values are 1.00 and 3.20.[25] Thus at the 1 percent level the observed values of R do not differ significantly from 2.10, and the hypothesis that the computed residuals from the reduced form equations are random has not been discredited.[26]

The R values for the structural equations are:

Demand for	R
pork	2.62
beef	2.64
poultry products	2.57

Thus the hypothesis that the residuals in the structural equations are random is not discredited.

The likelihood ratio test has been applied to each of the three meat equations. This test involves forming the quantity $V = T \log (1 + v)$. The V values are as follows:

pork	0.0008129954
beef	0.0000402323629
poultry	0.000135280095

Since D exceeds $H - 1$ by only 1, we want the critical values of chi-square for 1 degree of freedom in each case. The critical value is 3.841 at the 5 percent level of significance. Since all the test quantities are very near zero, the hypothesis that the three equations are uniquely determined is not overthrown.[27]

However, the results of this model conflict with our assumption in several striking details. In both the pork and beef equations, we find that consumption decreases with an increase of current income. Also the sign of β_{34} conflicts with our assumption about it; it has been assumed that the high consumption of poultry products is associated with a low price of poultry products. Either the form of the equations comprising the model is wrong, the assumption about the effect of poultry prices on poultry consumption is wrong or some of the general assumptions of the model (e.g., the assumption that the residuals are not autocorrelated) are wrong.

[25] B. S. Hart and J. von Neumann, Tabulation of the probabilities for the ratio of the mean square successive difference to the variance. 13 Annals of Mathematical Statistics 207, June 1942.

[26] As has been suggested above, the test is valid for disturbances rather than residuals; but the disturbances are not observed.

[27] In the matter of assumptions this must be realized if this test is to be considered valid, see footnote 19, page 1002. It should be noted that the V values are unusually low.

Three other details of the results appear strange, though they do not violate any assumptions included in the model. The elasticity of beef consumption with respect to beef price (—3.34) seems to be higher than was to be expected. Also the pork equation indicates that the consumption of pork is low when the prices of beef and poultry products are high, and the beef equation indicates that the consumption of beef is low when the price of poultry products is high. These results indicate rather strange substitution relations among the meats. Finally, the beef equation indicates that high beef consumption is associated with low levels of past income. (Of course, sampling errors may exert considerable influence on the results.)

It is possible to construct a confidence region for the estimates of the structural parameters if the model satisfies substantially the same collection of conditions necessary to validate the likelihood ratio test made above (footnote 17, page 1001). However, the gain from setting up the confidence region does not appear to be commensurate with the expense involved, and so it has not been constructed.

UNIQUELY DETERMINED MODEL

A second model has been constructed since some of the results of the first one seem dubious. In the overidentified model, the use of both time and time squared as variables may have introduced errors associated with multicollinearity. At the present time there is no theoretical basis for deciding how to handle a case where some of the "independent" variables in a least squares regression (such as those of the reduced form) are highly correlated. In the second model the variable z_2 (time squared multiplied by log e) has been omitted, since there seems to be reason to think that the correlation between z_1 and z_2 may be excessively high.

The second model has been made uniquely determined by dropping the variable z_{10}, an index of the logarithms of one-year-lagged feed grain yield per acre. This variable appeared only in the feed grain production equation. If the logic of the situation suggested that z_{10} is very strongly significant in the feed grain production equation, then dropping the variable would significantly damage the model. However, there are many cases where a variable is thought to have a rather minor influence *a priori*. Then considerations of convenience can properly determine whether the variable is included in the analysis. The case of z_{10} appears to fall in this category.

Dropping z_2 alone would not have affected uniqueness, since it appeared in all the equations of the original model. It should be noted, however, that in the present case the difference between the two models is due partly to the removal of z_{10}, and partly to a change in the degree to which multicollinearity problems enter into the handling of the model.

In the uniquely determined model the three meat demand equations are:

(90) Pork

$$\begin{aligned} y_1 = &- 0.91y_2 + 0.60y_3 + 0.87y_4 - 1.23y_5 \\ &- 0.91y_6 - 0.03z_1 + 0.16z_3 + 0.76z_4 \\ &+ 0.29z_5 + 2.70 \end{aligned}$$

(91) Beef

$$\begin{aligned} y_7 = &\ 0.53y_2 - 0.77y_3 + 0.67y_4 - 0.22y_5 \\ &- 1.09y_6 - 0.02z_1 + 0.29z_3 + 0.65z_4 \\ &- 0.12z_5 + 3.06 \end{aligned}$$

(92) Poultry products

$$\begin{aligned} y_8 = &\ 0.12y_2 + 0.28y_3 - 0.68y_4 + 0.22y_5 \\ &+ 0.31y_6 + 0.002z_1 + 0.36z_3 + 0.53z_4 \\ &+ 0.28z_5 + 0.42 \end{aligned}$$

In this model pork and beef are complementary with dairy products and oleomargarine, while poultry products are substitutes for dairy products and oleomargarine. These relations are somewhat dubious, but probably not decisively so. There seems to be nothing else to make the model suspect. It appears superior to the over determined model in terms of reasonableness of results.

The von Neumann ratio test has been applied to both the reduced form equations and the structural equations of this uniquely determined model.[28] As in the case of the over-

[28]As has been stated earlier, the von Neumann test is strictly valid only for disturbances, rather than residuals.

determined model the most likely value of R is 2.10, the 5 percent critical values of R are 1.30 and 2.90 and the 1 percent critical values are 1.00 and 3.20. The results are as follows:

Dependent variable	R
y_1	2.44
y_2	2.40
y_3	1.72
y_4	2.14
y_5	2.30
y_6	2.48
y_7	1.74
y_8	2.05

Thus the hypothesis that the residuals from the reduced form equations are random is not discredited. For the structural equations we have

Demand for	R
pork	2.11
beef	2.24
poultry products	1.67

and thus hypothesis that the residuals in the structural equations are random is not discredited.

LEAST SQUARES METHOD

For purposes of comparison with the uniquely determined equations, a least squares equation has been set up for each of the three goods (pork, beef, poultry products) dealt with in the uniquely determined model. Two new exogenous variables have been included: z_{12}, the price of non-foods (derived from consumers price series by eliminating the influence of food items), and z_{13}, the price of foods excluding the three meat products and dairy products. The price of margarine has not been included in the demand equations. The grounds for its exclusion are *a priori*.[29] Thus z_{13} had to be substituted for z_3, the "other food" series that excluded the influence of margarine. Table 1 shows the comparison between the uniquely determined equations and the least squares equations.

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[29]In the simultaneous equation models there was some reason for including the margarine price in the meat demand equations, since it was already in the model in connection with the equations pertaining to dairy products.

TABLE 1. ESTIMATES OF REGRESSION COEFFICIENTS IN PORK, BEEF AND POULTRY PRODUCTS CONSUMPTION EQUATIONS

		y_2	y_3	y_4	y_5	y_6	z_1	z_3	z_4	z_5	z_{12}	z_{13}
Pork	Uniquely determined	—0.91	0.60	0.87	—1.23	—0.91	—0.03	0.16	0.76	0.29		
	Single equation	—0.78**	0.13**	0.002	0.31		—0.01**		0.43**	0.22	1.45	0.68
Beef	Uniquely determined	0.53	—0.77	0.67	—0.22	—1.09	—0.02	0.29	0.65	—0.12		
	Single equation	0.16	—0.96**	0.23	0.40		0.01**		0.33	—0.40*	0.72	0.38
Poultry products	Uniquely determined	0.12	0.28	—0.68	0.22	0.31	0.002	0.36	0.53	0.28		
	Single equation	—0.14	0.30	—0.46	—0.36		—0.02**		0.74**	0.51	0.74*	1.00

*Significant at the 5 percent level.
**Significant at the 1 percent level.

Confidence regions have not been calculated for the uniquely determined model for the same reason mentioned in connection with the overdeterminate model. But table 1 indicates which of the least squares estimates of parameters are significantly different from zero at the 1 and 5 percent levels. The prices of beef and pork are important in determining the quantity of pork but only the price of beef is important in determining the quantity of beef. Perhaps pork is consumed by people whose powers of digestion can cope with either pork or beef, while beef is eaten by some people who consider pork too greasy for them. Thus a fall in the price of beef will decrease pork consumption and increase beef consumption, but a fall in the price of pork will not have a strong tendency to induce people to substitute pork for beef.

It is slightly strange that the consumption of poultry products is not significantly influenced by any of the prices. Perhaps this can be explained partly by the fact that poultry products are standard fare for Thanksgiving and Christmas and the fact that some families serve poultry products on most Sundays. Current income is important for the consumption of pork and poultry products, but not beef. Past income is important for poultry products and fairly important for beef. The discrepancies in income effects among the various foods are not readily explainable, it seems. The price of other food is not important for any of the equations. The prices of non-food items are fairly important for poultry products, but not for pork or beef. The price of dairy products is not important in any equation.

It is particularly interesting that time is the only variable that is significant (1 percent level) in all three equations. This fact is also somewhat disturbing. As a first approximation, we may say that there has been a significant change of taste over the period surveyed. Strictly we ought to mean by this that no factor external to the individual consumer has brought about the change in his purchases. But it is also possible that some other series that ought to have been included in the analysis has been omitted. At the present time there is no indication which of these two explanations is more nearly correct.

Comparison of individual parameters in the two kinds of equations shows considerable deviation. The coefficients showing the influence of the price of dairy products are particularly extreme, since the two models give opposite signs in all three equations. However, it should be remembered that the least squares coefficient is not significant. Probably the price of dairy products has only a negligible influence on meat consumption; if this is true then it is not disturbing to find estimates with opposite signs. The same comment seems appropriate in connection with the fact that we have opposite signs for the two estimates of the elasticity of poultry consumption with respect to the price of pork, although it is surprising to find that this coefficient seems to be non-significant.

The greatest difficulty appears in connection with the time variable. It is highly significant in the least squares model, and yet the signs are opposite for the two models in both the beef and poultry cases. The inclusion of the price of non-food in the least squares equation represents the only significant difference between the lists of variables in the two cases. Since its coefficients are not highly significant, it seems unlikely that it can account for the difference of signs between the two sets of equations.

PREDICTIONS

A partial indication of the relative merits of the two systems is provided by comparing their abilities to predict meat consumption in 1947 and 1948, the first two years after the abandonment of price controls. Table 2 shows the predictions and the actual figures, all in logarithms. In this table one thing is particularly noteworthy. In each case, the closer prediction is made by the equation whose time coefficient had the smaller absolute value. This makes it look as though having a large time coefficient were an indication of inadequacy of the list of variables used in an equation.

TABLE 2. REALIZED PER CAPITA CONSUMPTION, COMPARED WITH PREDICTIONS FROM THE LEAST SQUARES EQUATIONS AND THE UNIQUELY DETERMINED EQUATIONS (IN LOGARITHMS).

		Realized	Predicted	
			Least squares	Just-identified
1947	pork	1.81224	1.71112	1.59223
	beef	1.73719	1.60004	1.47793
	poultry products	1.86982	1.89925	1.89396
1948	pork	1.80550	1.73455	1.64478
	beef	1.69984	1.56068	1.44618
	poultry products	1.86629	1.90376	1.87128

The predictions in table 2 are not very close to the realized figures. However, it should be noted that the test of the predictions is a severe one; the interwar figures are used as the basis for predicting 1947 and 1948 quantities.

In general, the least squares equations predict a little better than the uniquely determined equations, despite the strong chance that there has been a structural change between 1941 and 1948. The nature of least squares is such that the least squares prediction must be optimum within the period from which we get data used in fitting the regression; but there is no reason why this would have to happen when we use least squares to predict the future.

COMPARISON WITH OTHER STUDIES

It is interesting to observe the elasticity of the quantity of each meat with respect to the price of the same meat. The uniquely determined model shows the elasticities of pork and beef quantities with respect to their prices to be above 0.77 in absolute terms. These figures should be compared with the elasticity of the demand for meat with respect to the price of meat, as found in the studies of Shepherd, Tintner and French. Shepherd has fitted a least squares equation in which meat price is the dependent variable and per capita disposable income, per capita meat consumption and time are the independent variables.[30] The model is linear. Taking price and quantity at their averages, the elasticity of demand for meat with respect to its price is —0.75 for the period 1920 through 1941. It was to be expected that the elasticity of demand for meat would be less in absolute terms than the elasticity of demand for an individual kind of meat. It should be noted that the validity of this comparison is partly vitiated by the fact that the models involved differ considerably.

Tintner has used a uniquely determined shock model consisting of a demand equation and a supply equation to estimate the demand elasticity of meat with respect to its price.[31] The model is linear and covers the period 1919-41. In the demand equation the only variables are the price of meat, the per capita quantity of meat and the per capita disposable real income. At the averages of price and quan-

[30]Geoffrey Shepherd. Changes in the demand for meat and dairy products in the United States since 1910. Iowa Agr. Exp. Sta. Res. Bul. 368. 1949. pp. 384, ff.

[31]Gerhard Tintner. Static econometric models and their empirical verification, illustrated by a study of the American meat market. 2 Metroeconomica 3. 1951.

tity, the demand elasticity is —0.791. Tintner has also used an error model for the same data and finds the elasticity estimated at —0.818.

French obtained a strikingly different result for an overdetermined model covering 1919-41.[32] His demand equation for meat included the per capita consumption of meat, the retail price of meat, the prices of other foods, the prices of non-foods, per capita disposable income and time. On the basis of average prices and quantities, he found the price elasticity of the demand for meat to be —0.238.

It is interesting to compare our results with those Fox obtained for pork and beef.[33] Fox has used least squares equations that are linear in the first differences of the logarithms. Using first differences is a way of taking account of the trend influence. Fox has used the price as the dependent variable, but the reciprocal of the elasticity of price with respect to quantity is the elasticity of quantity with respect to price; it is also possible to solve his equations for quantity in order to get the elasticity of quantity with respect to income. Table 3 compares elasticities from Fox's equations with elasticities from the present uniquely determined model.

Both models are based on civilian consumption, and use the same period of years. The pork results agree well, considering the fact that Fox used first differences and the present study did not. The beef results show considerably less close agreement.

EGGS

SIMULTANEOUS EQUATIONS METHOD

OVERDETERMINED MODEL

The limited information method has also been used in deriving a demand curve for eggs at retail. All the equations are linear in the logarithms of the variables. The jointly dependent variables are as follows (no source is indicated for jointly dependent variables not appearing in the egg demand equation, since these variables do not appear in the calculations):

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[32]B. L. French, Application of simultaneous equations to the analysis of the demand for meat, unpublished M. S. thesis, Iowa State College, 1949. p. 41.

[33]Karl A. Fox. Factors affecting farm income, farm prices and food consumption, 3 Agricultural Economics Research 65, table 3, p. 71.

TABLE 3. ELASTICITY OF QUANTITY WITH RESPECT TO:

	Price		Income	
	Fox	Uniquely determined model	Fox	Uniquely determined model
Pork	0.86	0.91	0.77	0.76
Beef	0.94	0.77	0.83	0.65

y *The logarithm of:*

1. Per capita egg consumption, index number with 1935-39 = 100 (U. S. Bureau of Agricultural Economics, Outlook charts 1948, p. 41).
2. Retail price of eggs per dozen, index number with 1935-39 = 100, deflated by the consumer price index (U. S. Bureau of Labor Statistics, Handbook of labor statistics, 1947 edition, Bulletin 916, p. 121).
3. Retail price of meat, index number with 1935-39 = 100, deflated by the consumer price index of the Bureau of Labor Statistics (retail price of meat, ibid., p. 121).
4. Retail price of other foods, index number with 1935-39 = 100, deflated by the consumer price index of the Bureau of Labor Statistics (retail price of other foods, ibid., p. 121. The Bureau of Labor Statistics figures were adjusted to exclude meat and eggs).
5. Prices paid to farmers for eggs, index number with 1935-39 = 100, deflated by the consumer price index (1920 through 1923, U. S. Bureau of Agricultural Economics, Outlook charts 1947, p. 98; 1924 through 1941, U. S. Bureau of Agricultural Economics, Poultry ration costs and poultry feed price ratios, Mar. 1946, p. 20).
6. Per capita supply of eggs by farmers, index number with 1935-39 = 100 (U. S. Bureau of Agricultural Economics, Poultry and egg situation, Sept. and Oct. 1951, p. 11).

z *The logarithm of:*

1. Index number of per capita disposable income with 1935-39 = 100, deflated by the consumer price index (U. S. Bureau of Agricultural Economics, Consumption of food in the United States 1909-48, Misc. Pub. 691, Aug. 1949, p. 136).
2. Index of per capita disposable personal income lagged one year, with 1935-39 = 100, deflated by consumer price index (ibid., p. 136).
3. Time, in the form e^{at} (origin 1920).
4. Index of price paid to farmers for eggs lagged one year, with 1935-39 = 100, deflated by the consumer price index (1920-23 part of series, U. S. Bureau of Agricultural Economics, Outlook charts 1947, p. 98; after 1923, U. S. Bureau of Agricultural Economics, Poultry ration costs and poultry feed price ratios, p. 14).
5. Index of poultry ration cost per hundred pounds of ration, with 1935-39 = 100, deflated by consumer price index (U. S. Bureau of Agricultural Economics, Poultry ration costs and poultry feed price ratios, p. 14).
6. Index of poultry ration cost per hundred pounds of ration lagged one year, with 1935-39 = 100, deflated by the consumer price index (ibid., p. 14).

7. Index of cost of processing meat with 1935-39 = 100, deflated by the consumer price index (Bureau of Labor Statistics, Handbook of labor statistics, 1947 edition, Bulletin 916, p. 158).
8. Index of prices paid to farmers for meat lagged one year, with 1935-39 = 100, deflated by the consumer price index (Production and Marketing Administration, Livestock branch, Livestock market news 1947, July 1948, p. 76).
9. Index of cost of commodities used in the production of livestock with 1935-39 = 100, deflated by the consumer price index (U. S. Bureau of Agricultural Economics, Agricultural prices, Jan. 1950, p. 42).

The retail demand equations are as follows:

(93) Eggs

$$y_1 + \beta_{12}y_2 + \beta_{13}y_3 + \beta_{14}y_4 + \gamma_{11}z_1 + \gamma_{12}z_2 + \gamma_{13}z_3 = \zeta_1$$

(94) Meat

$$\beta_{22}y_2 + \beta_{23}y_3 + \beta_{24}y_4 + y_8 + \gamma_{21}z_1 + \gamma_{22}z_2 + \gamma_{23}z_3 = \zeta_2$$

(95) Other food

$$\beta_{32}y_2 + \beta_{33}y_3 + \beta_{34}y_4 + y_{11} + \gamma_{31}z_1 + \gamma_{32}z_2 + + \gamma_{33}z_3 = \zeta_3.$$

The consumers are represented as choosing the quantities of their purchases on the basis of relative prices and present income. In this model the only lagged income influence is the income of last year. As in the meats model, a time factor is included to reduce the danger of getting spurious correlations and to represent the influence of gradual social changes not explicitly introduced into the model.

The retail supply equations:

(96) Eggs

$$\beta_{41}y_1 + \beta_{42}y_2 + \beta_{44}y_4 + \beta_{45}y_5 + \beta_{46}y_6 + \gamma_{43}z_3 = \zeta_4$$

(97) Meat

$$\beta_{52}y_2 + \beta_{53}y_3 + \beta_{54}y_4 + \beta_{57}y_7 + \beta_{58}y_8 + \beta_{59}y_9 + \gamma_{53}z_3 = \zeta_5$$

(98) Other food

$$\beta_{62}y_2 + \beta_{63}y_3 + \beta_{64}y_4 + \beta_{6,10}y_{10} + \beta_{6,11}y_{11}$$
$$+ \beta_{6,12}y_{12} + \gamma_{63}z_3 = \zeta_6.$$

The retail supply of eggs is represented as depending on retail egg prices and prices to producers, the amount of eggs supplied by farmers and the time factor.

The demand equations in the commercial sector:

(99) Eggs

$$\beta_{72}y_2 + \beta_{75}y_5 + \beta_{76}y_6 + \gamma_{73}z_3 = \zeta_7$$

(100) Meat

$$\beta_{83}y_3 + \beta_{87}y_7 + \beta_{89}y_9 + \gamma_{83}z_3 + \gamma_{87}z_7 = \zeta_8$$

(101) Other food

$$\beta_{94}y_4 + \beta_{95}y_5 + \beta_{97}y_7 + \beta_{9,10}y_{10}$$
$$+ \beta_{9,12}y_{12} + \gamma_{93}z_3 = \zeta_9.$$

The farmers' supply functions:

(102) Eggs

$$\beta_{10,5}y_5 + \beta_{10,6}y_6 + \beta_{10,7}y_7 + \gamma_{10,3}z_3 + \gamma_{10,4}z_4$$
$$+ \gamma_{10,5}z_5 + \gamma_{10,6}z_6 = \zeta_{10}$$

(103) Meat

$$\beta_{11,3}y_3 + \beta_{11,5}y_5 + \beta_{11,7}y_7 + \beta_{11,9}y_9 + \beta_{11,10}y_{10}$$
$$+ \gamma_{11,3}z_3 + \gamma_{11,8}z_8 + \gamma_{11,9}z_9 = \zeta_{11}$$

(104) Other food

$$\beta_{12,4}y_4 + \beta_{12,5}y_5 + \beta_{12,7}y_7 + \beta_{12,10}y_{10} + \beta_{12,12}y_{12}$$
$$+ \gamma_{12,3}z_3 + \gamma_{12,10}z_{10} = \zeta_{12}.$$

It is assumed that the elasticity of the quantity of eggs with respect to the price of eggs is negative. No other assumption is made concerning individual parameters. In particular it is not assumed that the income elasticity of quantity is positive, since eggs may be an inferior good.

The coefficients are estimated for the retail demand for eggs equation. Since there are four endogenous variables in it, H is 4. There are three predetermined variables in it, and there are nine predetermined variables in the whole model; therefore D is 6. Since D is greater than H — 1, the model is overdetermined. The limited information method for overdetermined models gives the following equation for the retail demand for eggs:

$$(105)\quad y_1 = -0.58\,y_2 + 0.60\,y_3 - 0.49\,y_4 + 0.44\,z_1 + 0.29\,z_2 - 0.29\,z_3 + 1.71.$$

The only strange thing about this set of results is the coefficient of y_4, the price of other food. It appears reasonable to expect that raising the price of other food would increase the consumption of eggs; it seems unlikely that eggs are complementary to all other foods as a group. The results involve no conflict with the assumption that the elasticity of the quantity of eggs with respect to price is negative.

The likelihood ratio test has been used on this model also. D (the number of predetermined variables not appearing in the egg demand equation) is 6. H — 1 (where H is the number of jointly dependent variables in this equation) is 3. If the equation is to be considered uniquely determined, then the rank of π_{XY} must be H — 1. As in the case of the meat demand equations, we find the value of T log $(1 + v)$. This is 2.19156 for the egg demand equation. For 3 degrees of freedom (D — H — 1), the 5 percent critical value of χ^2 is 5.815. Thus we have not discarded the hypothesis that the model is uniquely determined.[34]

The von Neumann ratio test has been used on this overdetermined model. In the reduced form, the von Neumann ratios for the equations for the four jointly determined variables are as follows:

Equation	von Neumann ratio
y_1	2.22
y_2	2.17
y_3	2.04
y_4	1.59

In the basic demand equation for eggs the von Neumann ratio is 1.62. Since these calculations are based on the same

[34]For the assumptions that must be fulfilled if this test is to be valid, see footnote 17, page 1001.

number of observations as the calculations for the meats case, the 1 and 5 percent significance levels are the same as in the meats case; 1.00 and 3.20 at the 1 percent level, and 1.30 and 2.90 at the 5 percent level. All the above values of the von Neumann ratio are within the 5 percent level. Thus we have no reason to discard the hypothesis that the residuals from the reduced form equations and the egg demand equations are random.

LEAST SQUARES METHOD

Since the results of the overdetermined model appear to be reasonable, no other simultaneous model has been set up. However, a least squares single equation model has been set up using a demand equation in which the number of variable is greater than the number of variables in the egg demand equation of the overdetermined model. The new variables are:

z *The logarithm of*:

10. Index of retail price of dairy products with 1935-39 = 100, deflated by the consumer price index (price of dairy products, Bureau of Labor Statistics, Handbook of labor statistics, 1947 edition, Bul. 916, p. 121).
11. Index of retail price of all other foods with 1935-39 = 100, deflated by the consumer price index. Eggs, meat and dairy products are excluded (Bureau of Labor Statistics, op. cit.).
12. Index of retail price of non-food with 1935-39 = 100, deflated by the consumer price index (price of non-food items, Bureau of Labor Statistics, op. cit.).

It is assumed that the elasticity of egg quantity with respect to price is negative; no other assumption is made about the sign or magnitude of any coefficient.

The single equation is

$$\begin{aligned}(106)\quad y_1 = &-0.55\, y_2 - 0.12\, y_3 + 0.41\, z_1 + 0.27\, z_2 \\ &- 0.13\, z_3 - 0.27\, z_{10} - 0.18\, z_{11} - 1.40\, z_{12} \\ &+ 5.79.\end{aligned}$$

At the 5 percent level of significance only the coefficient of y_2 and z_1 are significant. It appears somewhat strange that the coefficients of all the prices are negative; this result makes eggs seem to be complementary with all other goods. Clearly this cannot be strictly true. It should be noted that the elasticity of egg quantity with respect to egg price is negative, however, as the model assumes it to be.

TABLE 4. COMPARISON OF COEFFICIENTS OF OVERDETERMINED AND SINGLE EQUATION MODELS.

Variable	Coefficient	
	Overdetermined model	Single equation model
y_2	—0.58	—0.55
y_3	0.60	—0.12
z_1	0.44	0.41
z_2	0.29	0.27
z_3	—0.29	—0.13

The value of the von Neumann ratio for the single equation is 1.61. As in the case of the overdetermined equation, the 1 percent critical values of the ratio are 1.00 and 3.20, and the 5 percent critical values are 1.30 and 2.90. Thus there is not adequate reason for discarding the hypothesis that the residuals from the single equation are random.

COMPARISON OF PARAMETERS

Table 4 enables us to compare the coefficients of the variables common to the two models.

Since "other food" does not have the same meaning in the two models, we cannot directly compare the coefficient of y_4 of the overdetermined model (—0.49) with the coefficient of z_{11} of the single equation model (—0.18). It is notable that the two models agree well on the values of the only coefficients (those of y_2 and z_1) that were significant at the 5 percent level in the single equation model. The coefficient for the retail price of meat shows the greatest difference between the models; in this case the result of the overdetermined model seems to be more reasonable.

PREDICTIONS

The choice between the overdetermined model and the single equation model is not entirely clear, since the single equation model includes three variables not included in the overdetermined model. We may have a case where the greater completeness of the single equation is more important than the clearer logical relation between the overdetermined demand equation and the rest of the economy.

The predictive ability of the two equations can be compared by seeing how nearly they predict the logarithms of the actual consumption in the postwar years 1947 through 1950. Table 5 shows the relevant figures.

TABLE 5. COMPARISON OF ESTIMATED AND ACTUAL LOGARITHMS OF CONSUMPTION.

	Actual per capita consumption	Estimated per capita consumption	
		Overdetermined	Single equation
1947	2.10380	2.01819	2.05059
1948	2.11394	2.02542	2.05120
1949	2.10720	2.00774	2.04325
1950	2.12385	2.04929	2.06528

Both models underestimate consumption in each of the postwar years. In each case the single equation comes considerably closer to predicting the actual consumption. As in the case of the meat demand equations, superior predictive ability is associated with a relatively small coefficient for the time variable (z_3 in this case).

DISCUSSION OF METHODS

The discussion of least squares procedure suggests that there is a strong reason for choosing the simultaneous equations method rather than the least squares method. Thus working with a complete model appears superior to using the least squares method on one equation selected from that model. But in practice this is not likely to be the choice. Rather the choice is between using a given complete model and using a single equation selected from a larger and presumably more realistic model. For a given complete model, using a simultaneous equation method to estimate the behavior coefficients of one equation is more expensive than using the least squares procedure to minimize errors in the direction of one of the endogenous variables of this equation. Thus, if we have a given amount of money to spend on a project (and all practical work is subject to such limitations) and if this amount of money is enough to permit us to use the limited information method, then the same amount of money would permit us to augment the number of variables to be used in the one equation we select for least squares treatment—say the equation explaining the demand for a single good.

It appears to be impossible at the present time to generalize about the relative advantages of simultaneous equations methods and the single equation least squares method. If we have good reason to think that only a few variables are important in a complete model to explain a set of economic events, then we may use the simultaneous method.

If on the other hand, we think that a reasonably complete model would include a large number of variables in the equation we are primarily interested in, we may decide to use a single equation method; we may decide that using only one equation of the adequate form of the complete model is preferable to keeping many of the equations while reducing the number of variables appearing in each of these equations.[35]

Parenthetically, there is no easy way of choosing among simultaneous equations models, nor of choosing among least squares models. In certain cases we can decide whether it is reasonable to add a variable to a given equation. But choosing between two types of equations is not a precise procedure.

The choice between a uniquely determined model and an overdetermined model is particularly interesting. If the models include the same number of equations and about the same number of variables, it appears that the computational cost of the overdetermined model will be about five times that of the uniquely determined model.[36] Assume that no computations have been made. If the investigator is unable to decide between the two models on *a priori* grounds, the cost element provides a conclusive answer. Even if the *a priori* considerations suggest a slight preference for the overdetermined model, it may still be prudent to use a uniquely determined model or models. After the coefficients of any model have been estimated some of them may violate assumptions the investigator has made before beginning calculations. Using a uniquely determined model makes it relatively inexpensive for the investigator to profit immediately from any insights that he may get from the first model used. Given the amount of money to be spent on an economic investigation, being able to experiment with several models may be more important than being able to use an overdetermined model.

While it seems impossible to decide in general whether a least squares model or a simultaneous model is superior, we may be able to decide in individual cases. In the meats problem it seems that greater reliance should be placed on

[35]A method of Bentzel and Wold (on statistical demand analysis from the standpoint of simultaneous equations, 29 Skandinavisk Aktuarietidskrift 95, 1946) offers considerable promise for future applications. If it is possible to find one equation involving only one current endogenous variable, then a second equation involving only one additional endogenous variable, and so on, then it is legitimate to use least squares throughout. Bentzel and Wold apply their method to a monopoly problem. Tintner has used the method in studying the price of corn. (See Gerhard Tintner, op. cit., pp. 276-7.)

[36]This is a very rough estimate based on the computational costs involved in the models in the present study. The ratio would certainly depend on the size range of the models worked with. With a small number of variables and equations the ratio might also be small.

the results of the simultaneous equation model than on the results of the least squares model. Essentially the only advantage of the least squares model is that it permits us to use more variables. But in the meats case it seems there are only a few variables that seem to have logical connections with the problem. Only the price of non-food items was added in going from the simultaneous to the least squares model.

But in the egg problem the least squares model seems preferable, since it predicts the 1947-50 results better than does the simultaneous model. Revisions of both models might reverse this judgment.

Even under the most favorable circumstances it is difficult to decide whether an experiment has been crucial with respect to the acceptance or rejection of a hypothesis. If the experiment produces the results predicted by the hypothesis, there is always the chance that some other hypothesis would have done just as well.

Moreover, whenever a continuous variable or a variable with small gradations is involved in an economic problem, it is hard to decide how large the errors of prediction can be without discrediting the hypothesis.

In economics the situation is particularly discouraging because conditions change fast enough so that checks of relative accuracy of prediction under two hypotheses are subject to sampling variation over time. If the first hypothesis predicts better than the second during the first 2 years of the prediction period, we may not be reasonably sure that it will maintain its advantage for the next 2 years.

We seem to be forced to decide whether a given hypothesis gives us a workable tentative basis for action pending possible further formulation and testing of hypotheses. It appears that the hypothesis represented by the uniquely determined model is preferable in the meats case. It appears to provide a reasonable basis for policy decisions—again, pending further investigation. It also seems to provide a reasonable starting point for further refinement of hypotheses.

It appears that the same statements can be made about the least squares model in the egg demand problem.

IMPLICATIONS FOR PROBLEMATIC SITUATIONS

PORK, BEEF AND POULTRY PRODUCTS

With reference to the uniquely determined model for the meat products, table 6 shows some of the most important elasticities.

TABLE 6. SUMMARY OF IMPORTANT ELASTICITIES.

Demand equation	Elasticity of quantity with respect to the price of:		
	Pork	Beef	Poultry products
Pork	—0.91	0.60	0.87
Beef	0.53	—0.77	0.67
Poultry products	0.12	0.28	—0.68

Demand equation	Elasticity of quantity with respect to:	
	Current income	Weighted average of incomes of past 5 years
Pork	0.76	0.29
Beef	0.65	—0.12
Poultry products	0.53	0.28

The elasticity of each meat quantity with respect to its own price is less than 1. This fact is significant in connection with price support operations. When the price of a product is raised there is an increase in the gross income of the farmers producing it. Since there is presumably some positive marginal cost of production, there will also be an increase in net income. Together with comparable cost information, the price elasticities help to predict the net income effect of any given price increase. It is conceivable that the basic intention of the voters is that of supporting net farm income through supporting farm prices; if this is true a price support level can reasonably be chosen largely on the basis of its expected effect on net income.

Separate price control actions in relation to individual meats are likely to change the relative prices of the meats. In planning a price support program, presumably an administrator would be interested in the cross elasticities of demand. Apparently changes in the prices of pork and beef have little effect on the consumption of poultry products, but changes in the price of poultry products have somewhat larger effects on the consumption of pork and beef. When the price of pork changes, the effect on the consumption of beef is more important than the effect on the consumption of dairy products; and when the price of beef changes, the effect on the consumption of pork is more important than the effect on the consumption of poultry products. Considering all the price elasticities, we can conclude that adjustments in the prices of the various meats are likely to leave poultry consumption more stable than either pork or beef consumption.

When the price support administrator looks ahead to his activities in a coming period, he knows that the scale of his operations will depend partly on the national income. If the national income is expected to rise, no action in support of prices may be needed; if the national income is expected to fall very sharply, perhaps it will not be feasible to store a given good on the scale that would be necessary to maintain the original support price level.

The effects of lagged income appear to be relatively slight; moreover they are made suspect by the fact that the beef elasticity is negative. Therefore we shall consider just the elasticities with respect to current income. All the meats have relatively inelastic demands with respect to income. This means that the price support administrator will not receive much help, relatively, from an income increase. It also means that a fall in income is relatively unlikely to render his storage activities infeasible. It is interesting to observe that the order of the size of elasticities is the same for both income elasticities and price elasticities.

EGGS

In the case of the demand for eggs, the elasticity of quantity with respect to price is 0.55 in the least squares model. In the same model the elasticity of quantity with respect to income is 0.41. These elasticities might enter into the decision-making of a price support administrator in a way analogous to that outlined above for the corresponding elasticities in the meats equations.

NATURE OF CONCLUSIONS

The tentative nature of our conclusions must be emphasized strongly. The limitations of the data and the state of development of the theory both provide us with compelling reasons for making this statement. We have only annual figures, which cover up some of the economic relations germane to our problem. Apparently only very simple economic models are feasible at the present time because of expense.

It is also significant that price support decisions would have to rest in part on studies of cost conditions; only demand relations have been dealt with in the present study.

SUGGESTIONS FOR FURTHER STUDY

First, postwar data ought to be combined with the interwar data. Some serious problems arise in this connection. It may be that the postwar data follow functions quite different from those followed by the interwar data. But if this is true, then only the postwar data are strictly relevant for prediction of future figures.

It may be possible to study the reasons for the change in function between the interwar years and the postwar years. (Presumably data for years with rationing would be ambiguous.) But it might be still more helpful to find a function into which the interwar data and the postwar data could all be fitted.

Second, the investigation of the problematic situation of a price administrator clearly requires the investigation of the supply conditions. Presumably production functions would have to be fitted for several kinds of foods.

Third, there ought to be an effort to get quarterly data. Using annual totals deprives us of considerable information. On the other hand, the problem of autocorrelation becomes more serious as the length of the period is reduced. Perhaps it would be desirable to introduce quarterly data for some of the series, using for each quarter of other series either the annual figure or one-fourth of it, whichever would be appropriate. Some test of autocorrelation could be used to determine when the breakdown into quarterly data had gone far enough to cause significant autocorrelation of the residuals.

Fourth, some idea of the usefulness of the fairly complicated contemporary procedures might be gained by comparison of their predictions with the predictions of the so-called "naive models,"[37] which predict the coming year's consumption as being some simple function of this year's consumption, or both this year's consumption and last year's consumption. It is possible that the stability of the demand functions is so slight that using fairly long time series is a hindrance rather than a help.

In the present case it would not be reasonable to use naive models in testing the ability of the prewar equations to predict postwar results. A reasonable test of this nature could be made only if ability to predict 1951 results, say, were compared for a naive model predicting continuation of 1950 results and a regression system using economically significant series for several years extending through 1950.

Fifth, it might be interesting to experiment with graphic methods. There are repeated references above to difficulties

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[37] See Carl Christ, A test of an econometric model for the United States, 1921-1947, in Conference on Business Cycles, National Bureau of Economic Research, New York. 1951. pp. 35, ff.

growing out of the limitations affecting the forms of equations. Using graphic methods would remove these limitations. Algebraic methods are superior to graphic methods in precision, of course; but this may be a relatively minor consideration in the present case.